EXTRACTA MATHEMATICAE Vol. **20**, Núm. 3, 219 – 232 (2005)

# Cohomology Ring of $n$-Lie Algebras

Mikołaj Rotkiewicz

*Institute of Mathematics, Polish Academy of Sciences,*
*Śniadeckich* 8, 00-956, *Warsaw, Poland*

*and*

*Institute of Mathematics, University of Warsaw,*
*Banacha* 2, 02-097 *Warsaw, Poland*

(Presented by J. Grabowski)

AMS *Subject Class.* (2000): 16B55, 17B70, 16E40 *Received February* 2, 2005

## Abstract

Natural graded Lie brackets on the space of cochains of $n$-Leibniz and $n$-Lie algebras are introduced. It turns out that these brackets agree under the natural embedding introduced by Gautheron. Moreover, $n$-Leibniz and $n$-Lie algebras turn to be canonical structures for these brackets in a similar way in which associative algebras (respectively, Lie algebras) are canonical structures for the Gerstenhaber bracket (respectively, Nijenhuis-Richardson bracket). This allows to define the corresponding cohomology operators and graded Lie algebra structures on the cohomology spaces in an uniform simple way by means of square zero elements.

## References

[1] Balavoine, D., Deformations of Algebras over a Quadratic Operad, *C*ont. Math. **202** (1997), 207-234.
[2] Daletskii, Yu. L., Takhtajan, L.A., Leibniz and Lie Algebra Structures for Nambu Algebra, *Lett. Math. Phys.* **39** (1997), 127–141.
[3] Filippov, V.T., $n$-Lie algebras, *Sibirsk. Math. Zh.* **26**(6) (1985), 126–140.
[4] Gerstenhaber, M., The cohomology structure of an associative ring, *Ann. of Math.* **78** (2) (1963), 267–288.
[5] Gautheron, P., Some remarks concerning Nambu Mechanics, *Lett. Math. Phys.* **37** (1996), 103–116.
[6] Grabowski, J. , Marmo, G., Jacobi structures revisited, *J. Phys. A. Math. Gen.* **34** (2001), 10975–10990.

[7] Ibáñez, R., de León, M., Marrero, J.C., Padrón, E., Leibniz algebroid associated with a Nambu-Poisson structure, *J. Phys. A***32** (1999), no. 46, 8129–8144, arXiv:math-ph/9906027.

[8] Ibáñez, R., de León, M., López, B., Marrero, J.C., Padrón, E., Duality and modular class of a Nambu-Poisson structure, *J. Phys.* **A 34** (2001), no. 17, 3623–3650, arXiv:math.SG/0004065.

[9] Lecomte, P., De Wilde, M., Formal deformations of the Poisson-Lie algebra of a symplectic manifold and star products. Existence, equivalance, derivations, Deformation hteory of algebras and structures, *NATO-ASI Series C* **247** (1988), Kluwer Academic Publishers, 897–960.

[10] Loday, J-L, "Dialgebras. Dialgebras and related operads", Lecture Notes in Math., 1763, Berlin, Springer, 2001.

[11] Loday, J-L., Pirashvili, T., Universal enveloping algebras of Leibniz algebras and (co)homology, *Math. Ann.* **296** (1993), 139–158.

[12] Nambu, Y., Generalized Hamiltonian mechanics, *Phys. Rev.* **D7** (1973), 2405–2412.

[13] Takhtajan, L.A., On fundation of the generalized Nambu mechanics, *Comm. Math. Phys.* **160** (1994), 295–315.