Uniqueness of Invariant Hahn-Banach Extensions

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Received October 19, 2006

Abstract: Let $\ell$ be a linear functional on a subspace $Y$ of a real linear space $X$ provided with a sublinear functional $p$ with $\ell \leq p$ on $Y$. If $G$ is an abelian semigroup of linear transformations $T : X \to X$ such that $T(Y) \subseteq Y$, $p(Tx) \leq p(x)$ and $\ell(Ty) = \ell(y)$ for all $T \in G$, $x \in X$ and $y \in Y$ respectively, then a generalization of the classical Hahn-Banach theorem asserts that there exists an extension $\tilde{\ell}$ of $\ell$, $\tilde{\ell} \leq p$ on $X$ and $\tilde{\ell}$ remains invariant under $G$. The present paper investigates various equivalent conditions for the uniqueness of such extensions and these are related to nested sequences of $p$-balls, a concept that has proved useful in recent years in dealing with such extensions. The results are illustrated by a variety of examples and applications.

Key words: Sublinear functionals, nested sequences of ($p$-) balls, invariant Hahn-Banach extensions.


References


