

## Solvability of a Recursive Functional Equation in the Sequence Banach Space $l^2$

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*Abstract:* The main aim of this paper is to study the existence of solutions of the following recursive functional equation

$$x(n) = f(n, x(n), x(n-1))$$

in the space  $l^2$ , under general assumptions. The main tools of our existence theorem are the characterization of the relatively compact sets in the space  $l^2$  and Schauder Fixed point theorem. Moreover, our functional equation has as particular cases some integral equations of Urysohn type. Finally, we present some examples where our theorem can be applied.

*Key words:* Recursive functional equations, Banach sequence space, Fixed-point theorem.

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