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Structural Properties of Banach and Fréchet Spaces Determined by the Range of Vector Measures

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Abstract: The major theme of this paper is the interaction between structural properties of Banach and Frechet spaces and the measure-theoretic properties of measures taking values in these spaces. The emphasis shall be on the geometric/topological properties of the range of vector measures, including mainly the issue involving localization of certain (distinguished) sequences in these spaces inside the range of vector measures with or without bounded variation. Besides a brief discussion of the properties determined by the range of a vector measure, the paper concludes with a list of problems belonging to this area which are believed to be open.

Key words: Range of a vector measure, bounded variation, nuclear map, Banach space.

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