

Convex Sets without Diametral Pairs

LIBOR VESELÝ*

*Dipartimento di Matematica, Università degli Studi, Via C. Saldini 50
20133 Milano, Italy, libor.vesely@unimi.it*

Presented by Pier L. Papini

Received March 18, 2009

Abstract: Let X be an infinite dimensional normed linear space. It is not difficult to see that arbitrarily near (in the Hausdorff metric) to the unit ball of X there exists a nonempty closed convex set whose diameter is not attained. We show that such sets are dense in the metric space of all nonempty bounded closed convex subsets of X if and only if either X is not a reflexive Banach space or X is a reflexive Banach space in which every weakly closed set contained in the unit sphere S_X has empty relative interior in S_X .

Key words: Diametral pair, bounded closed convex set, Hausdorff metric.

AMS *Subject Class.* (2000): 46B20.

REFERENCES

- [1] R. BOURGIN, “Geometric Aspects of Convex Sets with the Radon-Nikodým Property”, Lecture Notes in Mathematics, 993, Springer-Verlag, Berlin, 1983.
- [2] R.E. MEGGINSON, “An Introduction to Banach Space Theory”, Graduate Texts in Mathematics, 183, Springer-Verlag, New York, 1998.
- [3] M. SABABHEH, R. KHALIL, Remotality of closed bounded convex sets in reflexive spaces, *Numer. Funct. Anal. Optim.* **29** (9-10) (2008), 1166–1170.
- [4] L. VESELÝ, Any infinite-dimensional Banach space contains a convex body without diametral points for its Minkowski gauge, *Boll. Un. Mat. Ital. A (7)* **8** (1) (1994), 95–99.

*Research partially supported by the MIUR, Italy.