

Studies on Dendrites and the Periodic-Recurrent Property*

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Abstract: In this paper we evidence the interest of considering three outstanding examples of dendrites with different structures, dendrites F_ω , W and G^3 . When a dendrite X contains a topological copy of one of them, then it is derived important properties. For example, if X does not contain a topological copy neither F_ω nor W , then X is a tree. If X does not contain a topological copy of G^3 then we obtain that X verifies the Periodic-Recurrent Property (the PR Property) which for dendrites is relevant under the point of view of Topological Dynamics. As an application of the former results, we give a unified proof of the fact that compact intervals of the real line $[a, b]$ ($a \neq b$), arcs and trees have also the PR Property.

Key words: PR Property, dendrites, Gehman dendrite, continua.

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