

## Torsion Graph of Modules

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*Abstract:* Let  $R$  be a commutative ring and  $M$  be an  $R$ -module. We associate to  $M$  a graph denoted by  $\Gamma(M)$  called the torsion graph of  $M$ , whose vertices are the non-zero torsion elements of  $M$  and two distinct elements  $x, y$  are adjacent if and only if  $[x : M][y : M]M = 0$ . We investigate the interplay between module-theoretic properties of  $M$  and graph-theoretic properties of  $\Gamma(M)$ . Among other results, we prove that  $\Gamma(M)$  is connected and  $\text{diam}(\Gamma(M)) \leq 3$  for a faithful  $R$ -module  $M$ .

*Key words:* Torsion graph, multiplication modules, diameter of torsion graph.

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### REFERENCES

- [1] M.M. ALI, D.J. SMITH, Finite and infinite collections of multiplication modules, *Beiträge Algebra Geom.* **42**(2) (2001), 557–573.
- [2] D.F. ANDERSON, A. FRAZIER, A. LAUVE, P.S. LIVINGSTON, The zero-divisor graph of a commutative ring, II, in “Ideal Theoretic Methods in Commutative Algebra” (Columbia, MO, 1999), Lecture Notes in Pure and Appl. Math., Marcel Dekker, New York, 220, 2001, 61–72
- [3] D.F. ANDERSON, P.S. LIVINGSTON, The zero-divisor graph of a commutative ring, *J. Algebra* **217**(2) (1999), 434–447.
- [4] D.D. ANDERSON, M. NASEER, Beck’s coloring of a commutative rings, *J. Algebra* **159**(2) (1993), 500–514.
- [5] M.F. ATIYAH, I.G. MACDONALD, “Introduction to Commutative Algebra”, Addison-Wesley, Reading, MA, 1969.
- [6] A. BARNARD, Multiplication modules, *J. Algebra* **71**(1) (1981), 174–178.
- [7] I. BECK, Coloring of commutative rings, *J. Algebra* **116**(1) (1988), 208–226.
- [8] A. CANNON, K. NEUERBURG, S.P. REDMOND, Zero-divisor graphs of nearrings and semigroups, in “Nearings and Nearfields” (A. Kreuzer, M.J. Thomsen, Eds.), Springer, Dordrecht, 2005 189–200.
- [9] F.R. DEMEYER, T. MCKENZIE, K. SCHNEIDER, The zero-divisor graph of a commutative semigroup, *Semigroup Forum* **65**(2) (2002), 206–214.
- [10] R. DIESTEL, “Graph Theory”, Springer-Verlag, New York, 1997.

- [11] Z.A. EL-BAST, P.F. SMITH, Multiplication modules, *Comm. Algebra* **16** (4)(1988), 755–779.
- [12] SH. GHALANDARZADEH, P. MALAKOOTI RAD, Torsion graph over multiplication modules, *Extracta Math.* **24** (3) (2009), 281–299.
- [13] I. KAPLANSKY, “Commutative Rings”, The University of Chicago Press, Chicago-London, 1974.
- [14] S.P. REDMOND, The zero-divisor graph of a non-commutative ring, *Int. J. Commut. Rings* **1** (2002), 203–211. (in “Commutative Rings”, Nova Sci. Publ., Hauppauge, NY, 2002, 39–47.)