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Ordinal Ranks on Weakly Compact and Rosenthal Operators †

Kevin Beanland, Daniel Freeman

Department of Mathematics and Applied Mathematics, Virginia Commonwealth University, Richmond, VA 23284, *kbeanland@vcu.edu*

Department of Mathematics, The University of Texas at Austin, Austin, TX 78712-0257, *freeman@math.utexas.edu*

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Abstract: Using the Schreier families $(\mathcal{S}_\xi)_{\xi<\omega_1}$, we define subclasses of weakly compact and Rosenthal operators between two Banach spaces. These subclasses give rise to ordinal ranks defined on each ideal. We prove several results concerning the analytic properties of this rank and give examples of spaces on which the ranks are bounded and unbounded.

Key words: weakly compact operators, ordinal ranks

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