

The Commutant of an Operator with Bounded Conjugation Orbits and C_0 -Contractions

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Abstract: Let A be an invertible bounded linear operator on a complex Banach space, $\{A\}'$ the commutant of A and B_A the set of all operators T such that $\sup_{n \geq 0} \|A^n T A^{-n}\| < +\infty$. Equality $\{A\}' = B_A$ was studied by many authors for different classes of operators. In this paper we investigate a local version of this equality and the case where A is a C_0 -contraction.

Key words: Operators, commutant, bounded conjugation orbit, C_0 -contraction.

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