EXTRACTA MATHEMATICAE Vol. **27**, Núm. 2, 245 – 300 (2012)

Expression de la Différentielle d_3 de la Suite Spectrale de Hochschild-Serre en Cohomologie Bornée Réelle

A. Bouarich

Université Sultan Moulay Slimane, Faculté des Sciences et Téchniques,
B.P. 523, *Beni Mellal, Maroc/Morocco*
bouarich1@yahoo.fr or bouarich@fstbm.ac.ma

Presented by A. Martínez Cegarra — Received November 26, 2007 *

Abstract: For discrete groups, we construct two bounded cohomology classes with coefficients in the second space of the reduced real ℓ_1-homology. Precisely, we associate to any discrete group G a bounded cohomology class of degree two noted $\mathfrak{g}_2 \in H_b^2(G, \overline{H}_2^{\ell_1}(G,\mathbb{R}))$. For G and Π groups and $\theta : \Pi \to \mathrm{Out}(C)$ any homomorphism we associate a bounded cohomology class of degree three noted $[\theta] \in H_b^3(\Pi, \overline{H}_2^{\ell_1}(G,\mathbb{R}))$. When the outer homomorphism $\theta : \Pi \to \mathrm{Out}(C)$ induces an extension of G by Π we show that the class $\mathfrak{g}_2$ is Π-invariant and that the differential d_3 of Hochschild-Serre spectral sequence sends the class $\mathfrak{g}_2$ on the class $[\theta]$: $d_3(\mathfrak{g}_2) = [\theta]$. Moreover, we show that for any integer $n \geq 0$ the differential $d_3 : E_3^{n,2} \to E_3^{n+3,0}$ of Hochschild-Serre spectral sequence in real bounded cohomology is given as a cup-product by the class $[\theta]$.

Key words: Cohomology of groups, ℓ_1-homology of groups, bounded cohomology of groups, spectral sequences, Banach spaces.

AMS *Subject Class.* (2010): 20J06, 55T05, 46A22

Références

[1] C. Bavard, Longueur stable des commutateurs, *Enseign. Math. (2)* **37, no. 1-2** (1991), 109 – 150.

[2] G. Besson, "Sur la Cohomologie Bornée", Sém. de cohomologie bornée, E.N.S. de Lyon, 1988.

[3] A. Bouarich, "Suites Exactes en Cohomologie Bornée Réelle", Thèse, Université Paul Sabatier, Toulouse III, 1994.

[4] A. Bouarich, Suites exactes en cohomologie bornée réelle des groupes discrets,*C. R. Acad. Sci. Paris Sér. I Math.* **320, no. 11** (1995), 1355 – 1359.

[5] A. Bouarich, Exactitude à gauche du foncteur $H_b^n(-,\mathbb{R})$ de cohomologie bornée réelle, *Ann. Fac. Sci. Toulouse Math. (6)* **10, no. 2** (2001), 255 – 270.

*Conditionally accepted for publication on October 31, 2008. Final acceptance on December 1, 2010.

[6] A. Bouarich, Suites spectrales de Hochshild-Serre à coefficients dans un espace semi-normé, *Extracta Mathematicae* **20, no. 3** (2005), 307 – 340.

[7] A. Bouarich, Sur une classe de cohomologie bornée de degré deux universelle, en préparation.

[8] R. Brooks, Some remarks on bounded cohomology, *Ann. of Math. Stud.* **97** (1981), 53 – 63.

[9] H. Brezis, "Analyse Fonctionnelle. Théorie et Applications", Masson, Paris, 1983.

[10] K.S. Brown, "Cohomology of Groups", Springer-Verlag, New York, 1982.

[11] Greenleaf, "Invariant Means on Topological Groups and their Applications", Van Nostrand Mathematical Studies No. 16, Van Nostrand Reinhold Co., New York-Toronto, Ont.-London, 1969.

[12] M. Gromov, Volume and bounded cohomology, *Publ. Math. IHES* **56** (1982), 5 – 99.

[13] A. Grothendieck, "Produits Tensoriels Topologiques et Espaces Nucléaires", Mem. Amer. Math. Soc. 1955, no. 16, Providence, 1955.

[14] A. Guichardet, "Cohomologie des Groupes Topologiques et des Algèbres de Lie", Cedic-Nathan, Paris, 1980.

[15] N. Ivanov, Foundations of the theory of bounded cohomology, *J. Soviet Math.* **37** (1987), 1090 – 1115.

[16] N. Ivanov, Second bounded cohomology group, *J. Soviet Math.* **52, No.1** (1990), 2822 – 2824.

[17] S. Maclane, "Homology", Springer-Verlag, Berlin, 1963.

[18] S. Matsumoto, S. Morita, Bounded cohomology of certain groups of homeomorphisms, *Proc. Amer. Math. Soc.* **94, no. 3** (1985), 539 – 544.

[19] J. McCleary, "User's Guide To Spectral Sequence", Mathematics Lecture Series, 12. Publish or Perish, Inc., Wilmington, DE, 1985.

[20] Y. Mitsumatsu, Bounded cohomology and l_1-homology of surfaces, *Topology* **23, no. 4** (1984), 465 – 471.

[21] N. Monod, M. Burger, Continuous bounded cohomology and applications to rigidity theory, *Geom. Funct. Anal.* **12, no. 2** (2002), 219 – 280.

[22] A. Noskov, Bounded cohomology of discrete groups with coefficients, *Leningrad Math. J.* **2, no. 5** (1991), 1067 – 1084.

[23] A. Noskov, The Hochschild-Serre spectral sequence for bounded cohomology, *Contemporary Mathematics* **131, Part 1** (1992), 613 – 629.

[24] T. Soma, The zero-norme subspace of bounded cohomology, *Comment. Math. Helv.* **72, no. 4** (1997), 582 – 592.

[25] K. Yosida, Functional Analysis, Springer-Verlag, Berlin, 1966.