

A MESOSCOPIC APPROACH FOR DIFFUSION IN EXPANDING MEDIA

Einstein's pioneering work on Brownian motion in 1905 laid the foundations of the modern description of diffusion transport processes in terms of random walks. The theory of random walks developed ever since has proven to be very successful to describe both normal and anomalous diffusion processes. However, the problem of diffusion in expanding media has received comparatively little attention so far in spite of the fact that a number of key biological processes take place in continuous growing media, notably protein migration in cells of morphogen gradient formation. In such cases, the average interparticle distance increases due to the medium expansion, resulting in a decrease of the encounter-controlled reaction rate. To account for such effects, the derivation of the standard evolution equation describing normal diffusion in static media must be conveniently generalized by switching to comoving coordinates [1]. Recently, this description has been extended to anomalous diffusion processes by means of a so-called continuous-time random-walk (CTRW) model [2]. For normal diffusion and a family of **superdiffusive processes (Lévy flights), the CTRW model yields exact expressions for the Green's function** (probability density of finding a random walker in a given volume element). As it turns out, the medium expansion/contraction enhances to a surprising extent differences between transport properties displayed by normal diffusive walkers and by walkers whose jumps are governed by a heavy-tailed waiting time distribution. **For example, the Green's function of a normal diffusive particle in an exponentially contracting medium tends to a broad stationary profile at long times. In contrast, a subdiffusive particle governed by a heavy-tailed CTRW becomes strongly localized at the origin in this long-time regime.**

References:

[1] S. B. Yuste, E. Abad, and C. Escudero, Diffusion in an expanding medium: Fokker-Planck equation, **Green's function and first-passage properties**, *Phys. Rev. E* 94, 032118 (2016).

[2] F. Le Vot, E. Abad, and S. B. Yuste, Continuous-time random-walk model for anomalous diffusion in expanding media, *Phys. Rev. E* 96, 032117 (2017).