

On Tauberian and Co-Tauberian Operators

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ABSTRACT

We show that a Banach space X has an infinite dimensional reflexive subspace (quotient) if and only if there exist a Banach space Z and a non-isomorphic one-to-one (dense range) Tauberian (co-Tauberian) operator from X to Z (Z to X). We also give necessary and sufficient condition for the existence of a Tauberian operator from a separable Banach space to c_0 which in turn generalizes a result of Johnson and Rosenthal. Another application of our result shows that if X^{**} is separable, then there exists a renorming of X for which, X is essentially the only subspace contained in the set of norm attaining functionals on X^* .

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