

Some Invariant Subspaces for A -Contractions and Applications

LAURIAN SUCIU

*Institut Camille Jordan, Université Claude Bernard Lyon 1,
69622 Villeurbanne cedex, France
e-mail: suciu@math.univ-lyon1.fr*

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ABSTRACT

Some invariant subspaces for the operators A and T acting on a Hilbert space \mathcal{H} and satisfying $T^*AT \leq A$ and $A \geq 0$, are presented. Especially, the largest invariant subspace for A and T on which the equality $T^*AT = A$ occurs, is studied in connections to others invariant or reducing subspaces for A , or T . Such subspaces are related to the asymptotic form of the subspace quoted above, this form being obtained using the operator limit of the sequence $\{T^{*n}AT^n; n \geq 1\}$. More complete results are given in the case when $AT = A^{1/2}TA^{1/2}$. Also, several applications for quasinormal operators are derived, involving their unitary, isometric and quasi-isometric parts, as well as their asymptotic behaviour.

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