Some Invariant Subspaces for $A$-Contractions and Applications

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Abstract

Some invariant subspaces for the operators $A$ and $T$ acting on a Hilbert space $H$ and satisfying $T^*AT \leq A$ and $A \geq 0$, are presented. Especially, the largest invariant subspace for $A$ and $T$ on which the equality $T^*AT = A$ occurs, is studied in connections to others invariant or reducing subspaces for $A$, or $T$. Such subspaces are related to the asymptotic form of the subspace quoted above, this form being obtained using the operator limit of the sequence $\{T^{*n}AT^n; \, n \geq 1\}$. More complete results are given in the case when $AT = A^{1/2}TA^{1/2}$. Also, several applications for quasinormal operators are derived, involving their unitary, isometric and quasi-isometric parts, as well as their asymptotic behaviour.

References


Key words and phrases: Invariant subspace, $A$-contraction, quasi-isometry, quasinormal operator