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# Substructures of Algebras with Weakly non-Negative Tits Form

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*Abstract*: Let $A = kQ/I$ be a finite dimensional basic algebra over an algebraically closed field $k$ presented by its quiver $Q$ with relations $I$. A fundamental problem in the representation theory of algebras is to decide whether or not $A$ is of tame or wild type. In this paper we consider triangular algebras $A$ whose quiver $Q$ has no oriented paths. We say that $A$ is essentially sincere if there is an indecomposable (finite dimensional) $A$-module whose support contains all extreme vertices of $Q$. We prove that if $A$ is an essentially sincere strongly simply connected algebra with weakly non-negative Tits form and not accepting a convex subcategory which is either representation-infinite tilted algebra of type $\tilde{E}_p$ or a tubular algebra, then $A$ is of polynomial growth (hence of tame type).

*Key words*: tame representation type, essentially sincere module, Tits form, strongly simply connected algebra.

AMS *Subject Class.* (2000): 16G20, 16G60, 16G70.

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