

Ball Proximality of Closed * Subalgebras in $C(Q)$

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Abstract: The notion of ball proximality and the strong ball proximality were recently introduced in [2]. We prove that a closed * subalgebra \mathcal{A} of $C(Q)$ is strongly ball proximal in $C(Q)$ and the metric projection from $C(Q)$, onto the closed unit ball of \mathcal{A} , is Hausdorff metric continuous and hence has continuous selection.

Key words: Proximinal, ball proximal, strongly ball proximal, metric projection, lower Hausdorff semi-continuity, upper Hausdorff semi-continuity, continuous selection.

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