

## Distances Between Composition Operators

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*Abstract:* Composition operators  $C_\varphi$  induced by a selfmap  $\varphi$  of some set  $S$  are operators acting on a space consisting of functions on  $S$  by composition to the right with  $\varphi$ , that is  $C_\varphi f = f \circ \varphi$ . In this paper, we consider the Hilbert Hardy space  $H^2$  on the open unit disk and find exact formulas for distances  $\|C_\varphi - C_\psi\|$  between composition operators. The selfmaps  $\varphi$  and  $\psi$  involved in those formulas are constant, inner, or analytic selfmaps of the unit disk fixing the origin.

*Key words:* Composition operators, norm–distance.

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