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Radial Projections of Bisectors in Minkowski Spaces

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Abstract: We study geometric properties of radial projections of bisectors in finite-dimensional real Banach spaces (i.e., in Minkowski spaces), especially the relation between the geometric structure of radial projections and Birkhoff orthogonality. As an application of our results it is shown that for any Minkowski space there exists a number, which plays somehow the role that $\sqrt{2}$ plays in Euclidean space. This number is referred to as the critical number of any Minkowski space. Lower and upper bounds on the critical number are given, and the cases when these bounds are attained are characterized. Some new characterizations of inner product spaces are also derived.

Key words: Birkhoff orthogonality, bisectors, characterizations of inner product spaces, critical number, isosceles orthogonality, Minkowski planes, Minkowski spaces, normed linear spaces, radial projection, Voronoi diagram.

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