EXTRACTA MATHEMATICAE Vol. **23**, Núm. 3, 235 – 242 (2008)

Range, Kernel Orthogonality and Operator Equations

Mohamed Amouch

Department of Mathematics, Faculty of Science, Semlalia
B.O. 2390 *Marrakesh, Morocco, m.amouch@ucam.ac.ma*

Presented by Manuel González — Received November 18, 2007

Abstract: Let $\mathcal{A}$ be a Banach algebra and $\mathcal{L}(\mathcal{A})$ the algebra of all bounded linear operators acting on $\mathcal{A}$. For $a, b \in \mathcal{A}$, the generalized derivation $\delta_{a,b} \in \mathcal{L}(\mathcal{A})$ and the elementary operator $\Delta_{a,b} \in \mathcal{L}(\mathcal{A})$ are defined by $\delta_{a,b}(x) = ax - xb$ and $\Delta_{a,b}(x) = axb - x$, $x \in \mathcal{A}$. Let $d_{a,b} = \delta_{a,b}$ or $\Delta_{a,b}$. In this note we give couples $(a, b) \in \mathcal{A}^2$ such that the range and the kernel of $d_{a,b}$ are orthogonal in the sense of Birkhoff. As application of this results we give consequences for certain operator equations inspired by earlier studies of the equation $\alpha + \alpha^{-1} = \beta + \beta^{-1}$ for automorphism α, β on Von Neuman algebras.

Key words: Elementary operators, orthogonality, operator equation.

AMS *Subject Class.* (2000): 47A20, 47B30, 47B47, 47B15.

"to my wife Hasna"

References

[1] E. Albrecht, P. G. Spain, When products of selfadjoints are normal, *Proc. Amer. Math. Soc.* **128** (2000), 2509 – 2511.

[2] F. F. Bonsall, J. Duncan, "Numerical Ranges of Operators on Normed Spaces and of Elements of Normed Algebras", London Math. Soc. Lecture Note Ser., 2, Cambridge Univ. Press, London-New York, 1971.

[3] C. K. Fong, Normal operators on banach space, *Glasgow Math. J.* **20** (1979), 163 – 168.

[4] P. R. Halmos, Commutators of operators, II, *Amer. J. Math.* **76** (1954), 191 – 198.

[5] R. C. James, Orthogonality and linear functionals in normed linear spaces, *Trans. Amer. Math. Soc.* **61** (1947), 265 – 292.

[6] D. C. Kleinecke, On operator Commutators, *Proc. Amer. Math. Soc.* **8** (1957), 535 – 536.

[7] J. Kyle, Numerical ranges of derivations, *Proc. Edinburgh Math. Soc.* **21** (1978/79), 33 – 39.

[8] A. M. Sinclair, Eigenvalues in the boundary of the numerical range, *Pacific J. Math.* **35** (1970), 231 – 234.