Range, Kernel Orthogonality and Operator Equations

MOHAMED AMOUCH

Department of Mathematics, Faculty of Science, Semlalia
B.O. 2390 Marrakesh, Morocco, m.amouch@ucam.ac.ma

Abstract: Let $A$ be a Banach algebra and $\mathcal{L}(A)$ the algebra of all bounded linear operators acting on $A$. For $a, b \in A$, the generalized derivation $\delta_{a,b} \in \mathcal{L}(A)$ and the elementary operator $\Delta_{a,b} \in \mathcal{L}(A)$ are defined by $\delta_{a,b}(x) = ax - xb$ and $\Delta_{a,b}(x) = axb - x$, $x \in A$. Let $d_{a,b} = \delta_{a,b}$ or $\Delta_{a,b}$. In this note we give couples $(a, b) \in A^2$ such that the range and the kernel of $d_{a,b}$ are orthogonal in the sense of Birkhoff. As application of this results we give consequences for certain operator equations inspired by earlier studies of the equation $\alpha + \alpha^{-1} = \beta + \beta^{-1}$ for automorphism $\alpha, \beta$ on Von Neuman algebras.

Key words: Elementary operators, orthogonality, operator equation.

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“to my wife Hasna”

REFERENCES