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On Orthocentric Systems in Strictly Convex Normed Planes

Horst Martini, Senlin Wu *

Faculty of Mathematics, Chemnitz University of Technology, 09107 *Chemnitz, Germany*
horst.martini@mathematik.tu-chemnitz.de

Department of Applied Mathematics, Harbin University of Science and Technology
150080 *Harbin, China, senlin_wu@hotmail.com*

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Abstract: It has been shown that the three-circles theorem, which is also known as Ţiţeica's or Johnson's theorem, can be extended to strictly convex normed planes, with various applications. From this it follows that the notions of orthocenters and orthocentric systems in the Euclidean plane have natural analogues in strictly convex normed planes. In the present paper (which can be regarded as continuation of [5] and [14]) we derive several new characterizations of the Euclidean plane by studying geometric properties of orthocentric systems in strictly convex normed planes. All these results yield also geometric characterizations of inner product spaces among all real Banach spaces of dimension ≥ 2 having strictly convex unit balls.

Key words: Birkhoff orthogonality, Busemann angular bisector, $\mathcal{C}$-orthocenter, inner product space, isosceles orthogonality, Minkowski plane, normed linear space, orthocenter, orthocentric system, three-circles theorem.

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