Convex Sets without Diametral Pairs

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Abstract: Let $X$ be an infinite dimensional normed linear space. It is not difficult to see that arbitrarily near (in the Hausdorff metric) to the unit ball of $X$ there exists a nonempty closed convex set whose diameter is not attained. We show that such sets are dense in the metric space of all nonempty bounded closed convex subsets of $X$ if and only if either $X$ is not a reflexive Banach space or $X$ is a reflexive Banach space in which every weakly closed set contained in the unit sphere $S_X$ has empty relative interior in $S_X$.

Key words: Diametral pair, bounded closed convex set, Hausdorff metric.


REFERENCES


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