

On the Berezin Symbols and Toeplitz Operators*

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Abstract: The present paper mainly gives some new applications of Berezin symbols. In particular, the Berezin symbol is used in approximation problem for H^∞ -functions. We study also asymptotic multiplicativity of the Berezin symbols. Moreover, we study the solvability of some Riccati operator equations of the form $XAX + XB - CX = D$ on the Toeplitz algebra \mathcal{T} , which is the C^* -subalgebra of the operator algebra $\mathcal{B}(L_a^2)$ generated by the Toeplitz operators $\{T_g : g \in H^\infty\}$ on the Bergman space $L_a^2(\mathbb{D})$. We characterize compactness of truncated Toeplitz operators $A_\varphi = P_{K_\theta} T_\varphi |_{K_\theta}$, $\varphi \in L^\infty(\mathbb{T})$, in terms of Berezin symbols. The spectrum of model operators $\varphi(M_\theta)$, $\varphi \in H^\infty$, is localized in terms of the so-called Berezin set by proving that $\sigma(\varphi(M_\theta)) \subset \text{closBer}(\varphi(M_\theta))$. Reducing subspaces of n -tuple of invertible operators on the Hilbert space H are described.

Key words: Berezin symbol, Hardy space, Bergman space, Toeplitz operator, truncated Toeplitz operator, inner function.

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