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Lattices and Manifolds of Classes of Flat Riemannian Tori

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Abstract: The topological and differentiable structures of some natural quotient spaces constructed from flat Riemannian tori are studied by means of a cut-and-paste procedure (concretely, $H\backslash(Gl^+(2,\mathbb{R})/Sl(2,Z))$, where $H = O^+(2,\mathbb{R})$, $CO^+(2,\mathbb{R})$, $O(2,\mathbb{R})$, $CO(2,\mathbb{R})$). In the orientation preserving cases, the quotients can be regarded as manifolds with singular points corresponding to lattices in the square and hexagonal crystal systems. In the non-orientation preserving ones, the natural structure is a smooth manifold with piecewise smooth boundary, where the interior points correspond to oblique lattices, the regular points of the boundary to rectangular and centered rectangular lattices and the edge of the boundary to square and hexagonal ones.

Key words: Flat and conformally flat tori, bidimensional crystallographic system, moduli space, lattice, orbifold.

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