

## Property (V) Still Fails the Three-Space Property

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*Abstract:* We construct twisted sums of  $C[0, 1]$  with itself in which the quotient map does not fix any copy of either  $C[0, 1]$  or  $c_0$ . We moreover show that every twisted sum of  $c_0(\Gamma)$  must have Pełczyński’s property (V).

*Key words:* Banach spaces, property (V), twisted sums.

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