

## Operator Valued Versions of the Radon-Nikodym Theorem and of the F. and M. Riesz Theorem \*

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Presented by Joe Diestel

Received August 3, 2012

*Abstract:* For an absolutely continuous operator valued measure in weak sense, we give a necessary and sufficient condition to have a density in strong sense. This result is used to obtain an operator valued version of the F. and M. Riesz theorem. We also give some related counterexamples.

*Key words:* Operator valued measure, weakly measurable, Fourier transform, absolutely continuous measure.

AMS *Subject Class.* (2010): 47A56, 46E27, 43A32.

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\*The first two authors were partially supported by the CDCH of the Universidad Central de Venezuela. The third author was partially supported by Dirección de Investigación y Postgrado de la UNEXPO.

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