INFLUENCE OF ROUGHNESS ON THE HYDRODYNAMIC DESCRIPTION OF A GRANULAR GAS

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Simple model of a granular gas: A collection of inelastic rough hard spheres

This model unveils an inherent breakdown of energy equipartition in granular fluids, even in homogeneous and isotropic states
Material parameters:

- Mass $m$
- Diameter $\sigma$
- Moment of inertia $I$ ($\kappa=4I/m\sigma^2$)
- Coefficient of normal restitution $\alpha$
- Coefficient of tangential restitution $\beta$
- $\alpha=1$ for perfectly elastic particles
- $\beta=-1$ for perfectly smooth particles
- $\beta=+1$ for perfectly rough particles
Energy collisional loss

\[ E_{ij} = \frac{1}{2} m v_i^2 + \frac{1}{2} m v_j^2 + \frac{1}{2} I \omega_i^2 + \frac{1}{2} I \omega_j^2 \]

\[ E'_{ij} - E_{ij} = -(1 - \alpha^2) \times \cdots \\
- (1 - \beta^2) \times \cdots \]

Energy is conserved \textit{only} if the spheres are
• elastic (\(\alpha=1\)) \textbf{and}
• either
  • perfectly smooth (\(\beta=-1\)) or
  • perfectly rough (\(\beta=+1\))
Elastic & smooth

Inelastic & (perfectly) rough

http://demonstrations.wolfram.com/InelasticCollisionsOfTwoRoughSpheres/
Outline of the talk

Granular temperatures, kurtoses, and correlations

**Translational temperature:** \( \langle v^2 \rangle = \frac{3T_t}{m} \)

**Rotational temperature:** \( \langle \omega^2 \rangle = \frac{3T_r}{I} \)

**Translational kurtosis:** \( \langle v^4 \rangle = \frac{5}{3} \langle v^2 \rangle^2 \left( 1 + a_{20}^{(0)} \right) \)

**Rotational kurtosis:** \( \langle \omega^4 \rangle = \frac{5}{3} \langle \omega^2 \rangle^2 \left( 1 + a_{02}^{(0)} \right) \)

**Scalar correlations:** \( \langle v^2 \omega^2 \rangle = \langle v^2 \rangle \langle \omega^2 \rangle \left( 1 + a_{11}^{(0)} \right) \)

**Angular correlations:** \( \langle (v \cdot \omega)^2 \rangle - \frac{1}{3} \langle v^2 \omega^2 \rangle \propto a_{00}^{(1)} \)
Boltzmann equation:

$$\partial_t f(r, v, \omega, t) + v \cdot \nabla f(r, v, \omega, t) = J(r, v, \omega, t|f)$$

Inelastic+Rough collisions
Theory (Sonine) vs simulations

\[ a_{11}^{(0)} \]
\[ a_{20}^{(0)} \]
\[ a_{00}^{(1)} \]

\[ a_{11}^{(1)} \]
\[ a_{20}^{(1)} \]
\[ a_{00}^{(2)} \]

\[ \alpha = 0.9 \]
\[ \alpha = 0.7 \]

\[ \theta \]
\[ \beta \]
Density plots
Conclusions (Part 1)

- The linearized Sonine approximation theory provides an excellent description of the temperature ratio and the four velocity cumulants, except when the angular velocity kurtosis becomes large ($a_{02}^{(0)} > 0.3$).
- The cumulants are relatively small in the experimentally relevant regime $\beta > 0$. 
Outline of the talk

Navier-Stokes-Fourier constitutive equations

Claude-Louis Navier (1785-1836)
George Gabriel Stokes (1819-1903)
Jean-Baptiste Joseph Fourier (1768-1830)
Navier-Stokes-Fourier constitutive equations

\[ P_{ij} = n T_t \delta_{ij} - \eta \left( \nabla_i u_j + \nabla_j u_i - \frac{2}{3} \delta_{ij} \nabla \cdot u \right) - \eta_b \delta_{ij} \nabla \cdot u \]

- Shear viscosity
- Bulk viscosity

\[ q = -\lambda \nabla T - \mu \nabla n \]

- Thermal conductivity

\[ \zeta = \zeta^{(0)} - \xi \nabla \cdot u \]

- Cooling rate transport coefficient
Boltzmann equation:

\[ \partial_t f(r, v, \omega, t) + \mathbf{v} \cdot \nabla f(r, v, \omega, t) = J[r, v, \omega, t \mid f] \]

Inelastic+Rough collisions
Methodology: Chapman-Enskog method

\[
f = f^{(0)} + \epsilon f^{(1)} + \epsilon^2 f^{(2)} + \cdots, \quad \epsilon \sim \nabla
\]

Sydney Chapman  
(1888-1970)

David Enskog  
(1884-1947)
### Special limiting cases

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Pure smooth ((\beta = -1))</th>
<th>Quasi-smooth limit ((\beta \to -1))</th>
<th>Perfectly rough and elastic ((\alpha = \beta = 1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\eta^*)</td>
<td>(\frac{24}{(1 + \alpha)(13 - \alpha)})</td>
<td>(\frac{24}{(1 + \alpha)(19 - 7\alpha)})</td>
<td>(\frac{6(1 + \kappa)^2}{6 + 13\kappa(1 + \kappa)^2})</td>
</tr>
<tr>
<td>(\eta_b^*)</td>
<td>0</td>
<td>(\frac{8}{5(1 - \alpha^2)})</td>
<td>(\frac{10\kappa}{10\kappa})</td>
</tr>
<tr>
<td>(\lambda^*)</td>
<td>(\frac{64}{(1 + \alpha)(9 + 7\alpha)})</td>
<td>(\frac{48}{25(1 + \alpha)})</td>
<td>(\frac{12(1 + \kappa)^2}{25(12 + 75\kappa) + 101\kappa^2 + 102\kappa^3})</td>
</tr>
<tr>
<td>(\mu^*)</td>
<td>(\frac{1280(1 - \alpha)}{(1 + \alpha)(9 + 7\alpha)(19 - 3\alpha)})</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(\xi)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Brey, Dufty, Kim, Santos (1998)  
Pidduck (1922)
Density plots
Conclusions (Part 2)

- Roughness induces two extra transport coefficients ($\eta_b$, $\xi$), not present in the case of a (dilute) gas of smooth spheres.
- Typically, at fixed $\alpha$ the coefficients have a maximum at an intermediate value of $\beta$.
- In general, the dependence of the coefficients on $\alpha$ is weaker than in the case of smooth spheres.
- Future application: Stability analysis of the HCS.
Thank you for your attention!