SEGREGATION IN GRANULAR MIXTURES: THERMAL DIFFUSION

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Segregation and mixing of dissimilar grains is one of the most interesting problems in vibrated granular mixtures

Several mechanisms have been proposed: Archimedean buoyancy, void filling, convection, frictional properties, ***thermal diffusion***,….

At large shaking amplitudes ⟶ Sample of grains resembles a **granular gas**

↓

Thermal diffusion: relevant *segregation* mechanism

Objective: segregation problem driven by the presence of a thermal gradient and the gravitational field

Thermal diffusion factor characterizes the amount of segregation parallel to the temperature gradient

Experimental conditions: inhomogenous *steady* state with zero flow velocity and zero mass flux

$$-\Lambda_{ij}\nabla \ln T = \frac{1}{x_i x_j}\nabla x_i, \quad \Lambda_{ij} + \Lambda_{ji} = 0,$$

Thermal diffusion factor

$$x_i = n_i/n$$

Binary granular mixture held between plates at different temperatures T’>T under gravity

Bottom plate is hotter than the top plate

$$\ln\left(\frac{x_1 x_2'}{x_2 x_1'}\right) = \Lambda_{12} \ln\left(\frac{T'}{T}\right)$$

Subscripts 1 and 2 refer to large and small particles

If $T' > T$ and $\Lambda_{12} > 0$, then $x_1' < x_1$ → BNE

If $T' > T$ and $\Lambda_{12} < 0$, then $x_1' > x_1$ → RBNE

Non-convecting steady state with only gradients in the vertical direction (z axis)

↓

Mass flux vanishes

Momentum blance equation ⟶ $\partial_z p = -\rho g$

$p = nT$

Constitutive equation for the mass flux

$$j_{1,z} = -\frac{m_1 m_2 n}{\rho} D \partial_z x_1 - \frac{\rho}{p} D_p \partial_z p - \frac{\rho}{T} D' \partial_z T$$

Garzó&Dufty, PF 2002

$$\Lambda_{12} = \frac{n\rho^2}{\rho_1\rho_2}\frac{D' - D_p g^*}{D}$$

$$g^* \equiv \rho g / n\partial_z T < 0$$

Determine the dependence of the thermal diffusion factor on the parameter space of the system (masses, sizes, composition, dissipation)

Model ⟶ *Low* density binary mixture of smooth **inelastic** hard spheres or disks

$$(m_i, \sigma_i, x_i, \alpha_{ij})$$

Kinetic theory tools $\longrightarrow$ Inelastic **Boltzmann** equation

$$\left(\partial_t + \mathbf{v}\cdot\nabla + \mathbf{g}\cdot\frac{\partial}{\partial \mathbf{v}} - \frac{1}{2}\frac{T_i}{m_i}\zeta_i\frac{\partial^2}{\partial \mathbf{v}^2}\right) f_i(\mathbf{r},\mathbf{v},t) = \sum_{j=1}^{2} J_{ij}\left[\mathbf{v}|f_i(t), f_j(t)\right]$$

gravity

white-noise thermostat

Barrat&Trizac, GM 2002; Dahl,Hrenya,Garzó&Dufty, PRE 2002

Chapman-Enskog expansion $\longrightarrow$ **Normal** or hydrodynamic solution to BE

To *first* order in the spatial hydrodynamic gradients

$$j_{1,z} = -\frac{m_1 m_2 n}{\rho} D \partial_z x_1 - \frac{\rho}{p} D_p \partial_z p - \frac{\rho}{T} D' \partial_z T.$$

Transport coefficients

First Sonine approximation: $D > 0, D' = 0$

$$\mathrm{sgn}(\Lambda_{12}) = \mathrm{sgn}(D_p)$$

$$D_p = \frac{\rho_1 p}{\rho^2 \nu} \frac{x_2}{x_2 + x_1 \gamma} \left(\frac{\gamma}{\mu} - 1 \right)$$

$$\mu = m_1/m_2$$

$$\gamma = T_1/T_2$$

(**positive**) collision frequency

Non-equipartition of energy

$$\theta \equiv \frac{\gamma}{\mu} = \frac{m_2 T_1}{m_1 T_2}$$

Mean-square velocity of the *large* particles relative to that of the *small* particles

Control parameter

If $\theta > 1 \quad (\theta < 1)$ → $\Lambda_{12} > 0 \quad (\Lambda_{12} < 0)$ → BNE (RBNE)

Criterion for **transition** → $\gamma = \mu$

Garzó, EPL 2006

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Due to the **lack** of energy equipartition: criterion is rather complicated since it involves *all* the parameter space of the system

$$\{m_1/m_2, \sigma_1/\sigma_2, x_1, \alpha_{11}, \alpha_{22}, \alpha_{12}\}$$

Similar results obtained by Trujillo et al. EPL 2003 (weak dissipation) and Brey et al. PRL 2005 (tracer limit)

Temperature ratio is determined from the condition

$$\gamma\zeta_1 = \mu\zeta_2 \quad \text{(white-noise thermostat)}$$

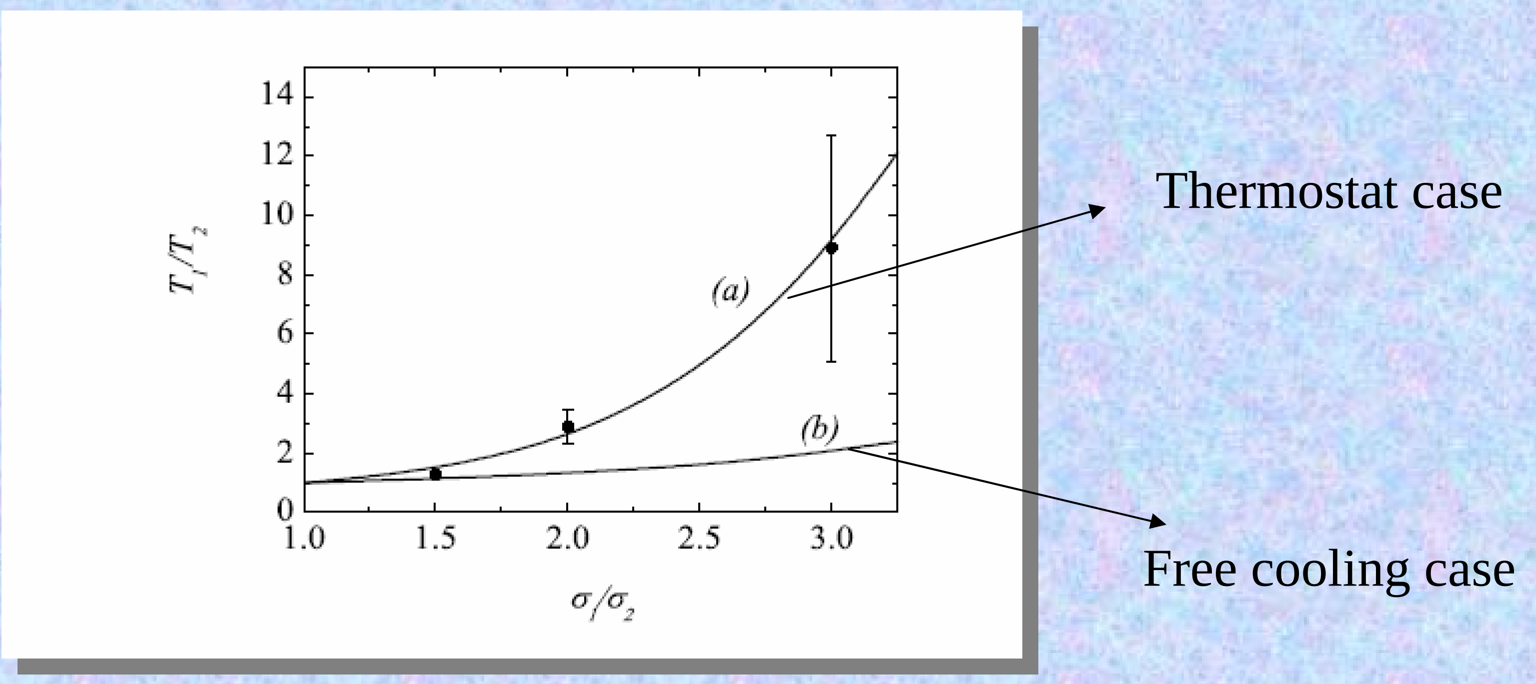
$$\zeta_1 = \zeta_2 \quad \text{(free cooling)}$$

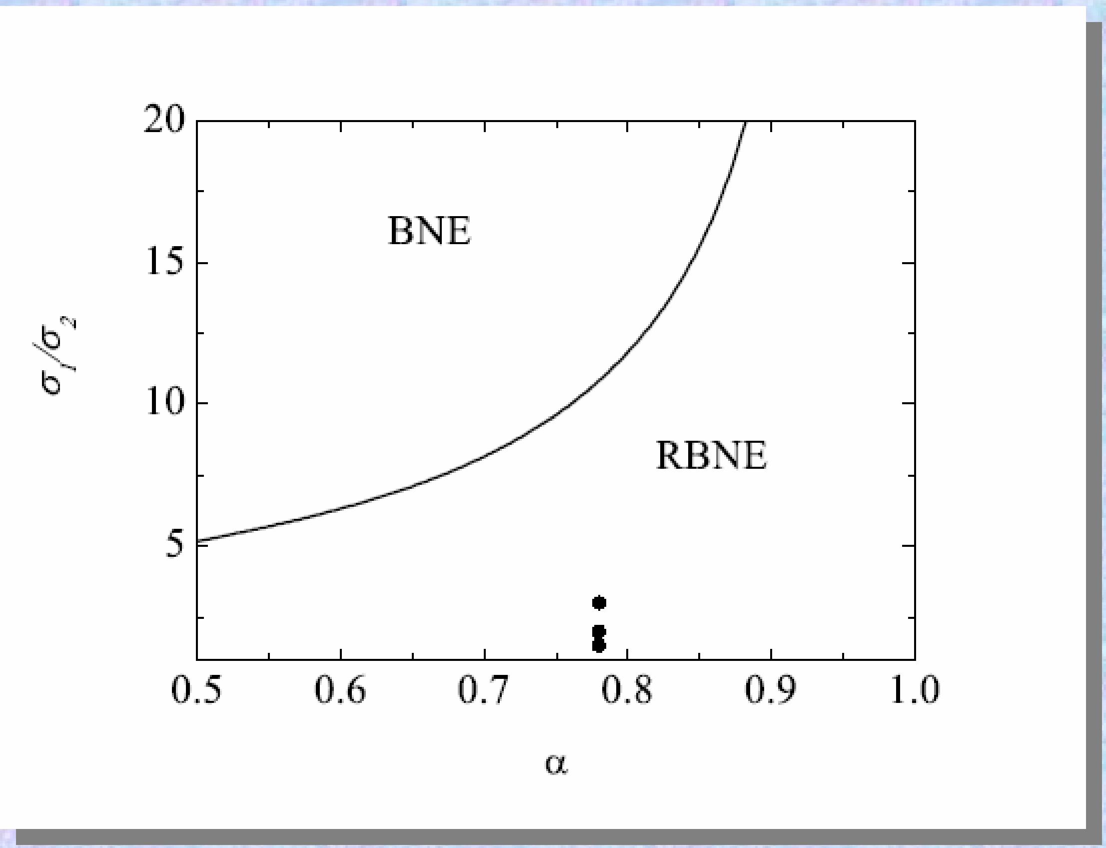
Brey et al. PRL2005

$$\zeta_i = \sum_{j=1}^{2} \zeta_{ij} = \frac{4\pi^{(d-1)/2}}{d\Gamma\left(\frac{d}{2}\right)} v_0 \sum_{j=1}^{2} n_j \mu_{ji} \sigma_{ij}^{d-1} \left(\frac{\theta_i + \theta_j}{\theta_i \theta_j}\right)^{1/2} (1+\alpha_{ij}) \left[1 - \frac{\mu_{ji}}{2}(1+\alpha_{ij}) \frac{\theta_i + \theta_j}{\theta_j}\right]$$

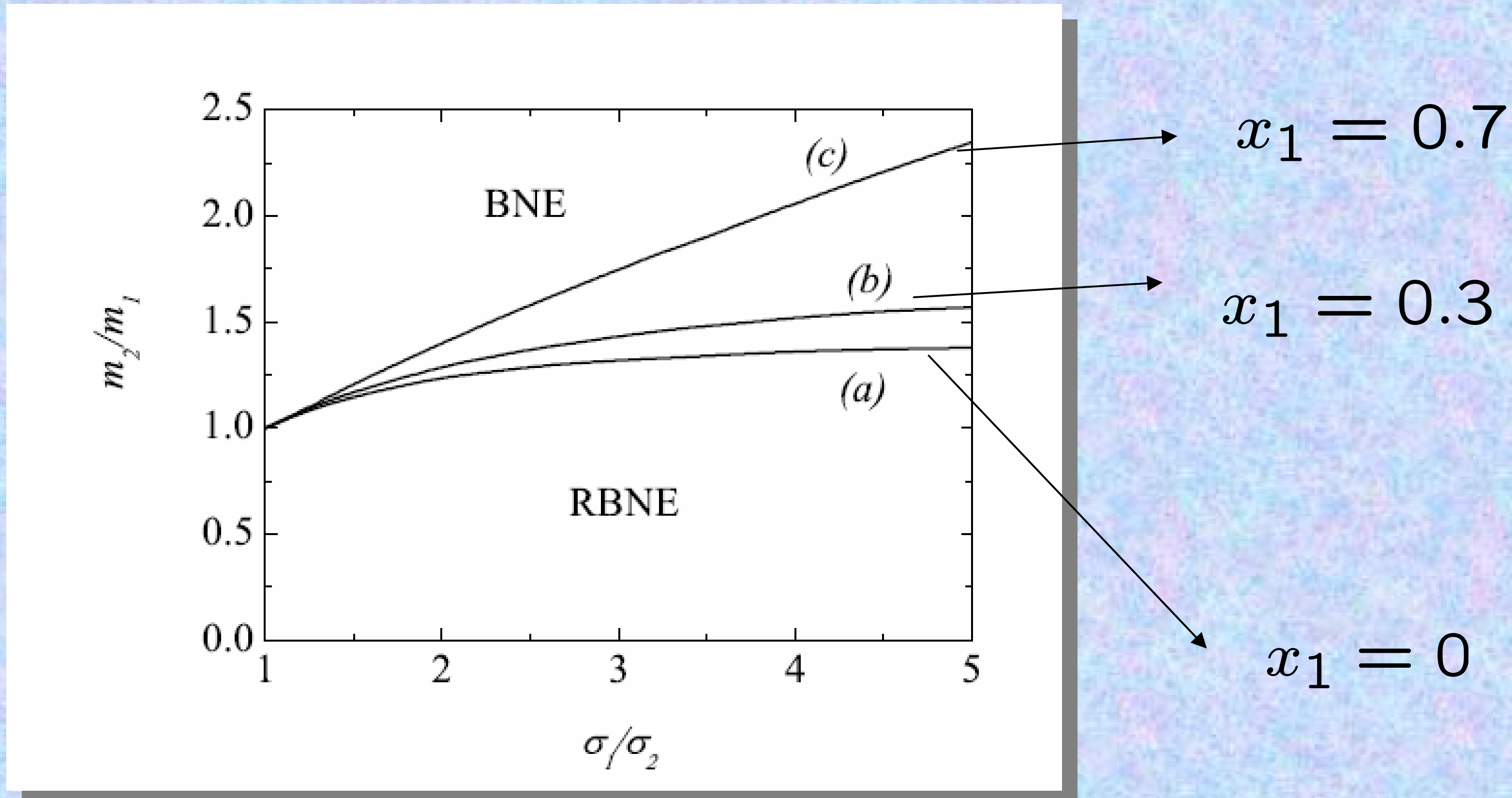
$$\theta_i = \frac{m_i}{\gamma_i} \sum_{j=1}^{2} m_j^{-1}, \quad \mu_{ij} = m_i/(m_i + m_j), \quad v_0(t) = \sqrt{2T(m_1 + m_2)/m_1 m_2}$$

Comparison with **experimental/simulation** results of Schroter, Ulrich, Kreft, Swift&Swinney (cond-mat/0601179, 2006) in *agitated mixtures* of particles of the same density





If one **extrapolates** from the simulation data, one sees that the transition from RBNE to BNE might be around 10



$$\alpha = 0.7$$

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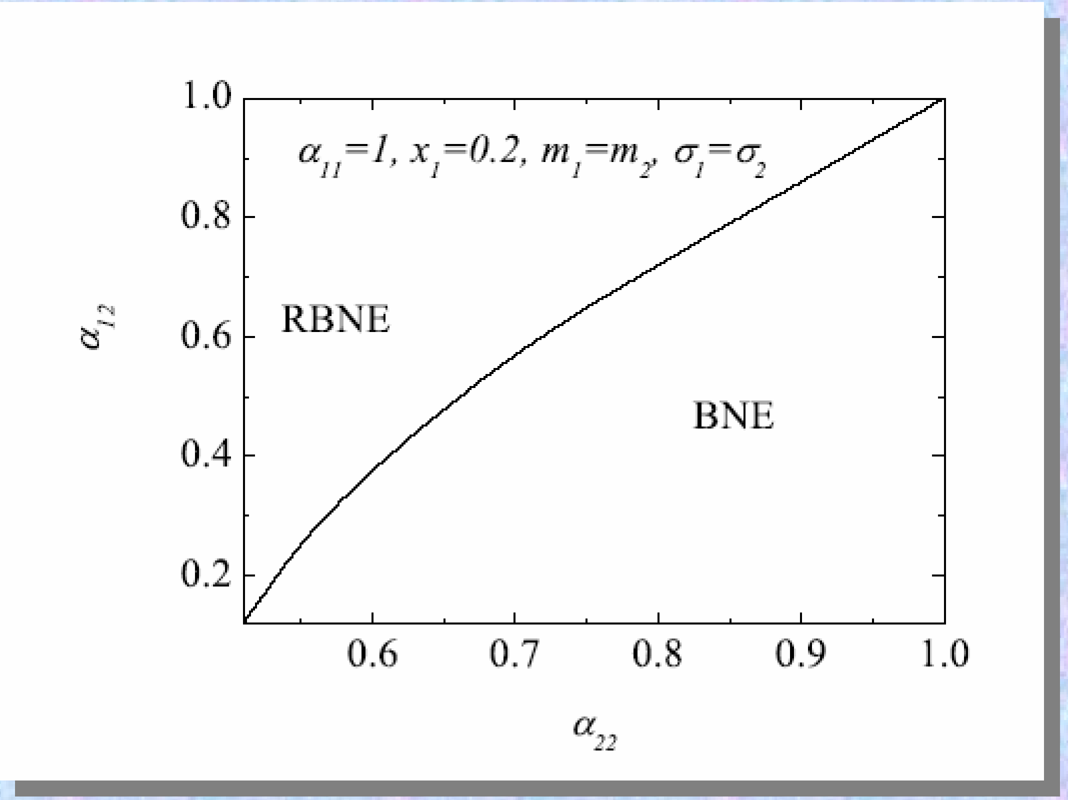

Mixture of *mechanically* equivalent particles

CONCLUDING REMARKS

- Segregation driven by a thermal gradient in agitated granular mixtures (Thermal diffusion)

- **Boltzmann kinetic theory** not restricted to weak dissipation and accounts for the non-equipartition of energy

- Good **qualitative** agreement with MD simulations, although a more quantitative comparison with simulations is needed in the low-density regime

Thanks for your attention