# RHEOLOGICAL PROPERTIES IN A LOW-DENSITY GRANULAR MIXTURE

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## INTRODUCTION

- Rapid granular flows → Fluid of hard spheres with *inelastic* collisions

- Simplest model: smooth hard spheres. Inelasticity characterized by a *constant* coefficient of normal restitution $\alpha$.

- Hydrodynamic equations obtained from kinetic theory. Low-density regime → *inelastic* Boltzmann equation. Dependence of the transport coefficients on dissipation in collisions.

- Most of the studies are devoted to the case of one-component systems. Much less is known on *multicomponent* granular systems. Usual assumption in prior works: Equal granular temperatures for all species. Weak dissipation approximation.

- Complexity increases in far from "equilibrium" states. Gradients can be controlled by dissipation and not only by boundary conditions. Simplest far from equilibrium physical situation: Simple (Uniform) Shear Flow. Uniform density and temperature and a *constant* velocity profile.

- **Objective**: Evaluation of the *rheological* properties in a binary granular mixture at low density in terms of inelasticity and mechanical parameters. Two complementary routes will be followed: a first-Sonine approximation and Monte Carlo simulations.

## BOLTZMANN EQUATION FOR A BINARY GRANULAR MIXTURE

- Binary mixture $\rightarrow \{m_1, m_2, \sigma_1, \sigma_2, \alpha_{11}, \alpha_{22}, \alpha_{12} = \alpha_{21}\}$.

- $\alpha_{ij}$ $\rightarrow$restitution coefficient for collisions $i - j$.

- Low-density regime: Distribution functions $f_i(\mathbf{r}, \mathbf{v}; t)$, $(i = 1, 2)$

$$(\partial_t + \mathbf{v}_1 \cdot \nabla) f_i(\mathbf{r}, \mathbf{v}_1, t) = \sum_j J_{ij} \left[\mathbf{v}_1 | f_i(t), f_j(t)\right] \ .$$

- Boltzmann collision operator $J_{ij}$

$$J_{ij} \left[f_i, f_j\right] = \sigma_{ij}^2 \int d\mathbf{v}_2 \int d\widehat{\sigma}\, \Theta(\widehat{\sigma} \cdot \mathbf{g}_{12})(\widehat{\sigma} \cdot \mathbf{g}_{12}) \left[\alpha_{ij}^{-2} f_i(\mathbf{v}_1') f_j(\mathbf{v}_2') - f_i(\mathbf{v}_1) f_j(\mathbf{v}_2)\right]$$

- $\sigma_{ij} = (\sigma_i + \sigma_j)/2$, $\mathbf{g}_{12} = \mathbf{v}_1 - \mathbf{v}_2$, $\mu_{ij} = m_i / (m_i + m_j)$

$$\mathbf{v}_1' = \mathbf{v}_1 - \mu_{ji} \left(1 + \alpha_{ij}^{-1}\right) (\widehat{\sigma} \cdot \mathbf{g}_{12}) \widehat{\sigma}, \quad \mathbf{v}_2' = \mathbf{v}_2 + \mu_{ij} \left(1 + \alpha_{ij}^{-1}\right) (\widehat{\sigma} \cdot \mathbf{g}_{12}) \widehat{\sigma}$$

- Decrease in the magnitude of the normal component of relative velocity at contact: $|\widehat{\sigma} \cdot \mathbf{g}_{12}| = \alpha_{ij} |\widehat{\sigma} \cdot \mathbf{g}_{12}'|$.

- The collision operators conserve particle number of each species and the total momentum but the total energy is not conserved:

$$\int d\mathbf{v}_1 J_{ij}[\mathbf{v}_1|f_i, f_j] = 0 \ ,$$

$$\sum_{i,j} \int d\mathbf{v}_1 m_i \mathbf{v}_1 J_{ij}[\mathbf{v}_1|f_i, f_j] = 0 \ ,$$

$$\sum_{i,j} \int d\mathbf{v}_1 \tfrac{1}{2} m_i v_1^2 J_{ij}[\mathbf{v}_1|f_i, f_j] = -\tfrac{3}{2} nT\zeta \ .$$

- These conditions lead to the macroscopic balance equations for the mixture:

$$D_t n_i + n_i \nabla \cdot \mathbf{u} + \frac{\nabla \cdot \mathbf{j}_i}{m_i} = 0 \ ,$$

$$D_t \mathbf{u} + \rho^{-1} \nabla \mathbf{P} = 0 \ ,$$

$$D_t T - \frac{T}{n} \sum_i \frac{\nabla \cdot \mathbf{j}_i}{m_i} + \frac{2}{3nT} \left( \nabla \cdot \mathbf{q} + \mathbf{P} : \nabla \mathbf{u} \right) = -\zeta T \ ,$$

- $D_t = \partial_t + \mathbf{u} \cdot \nabla$, $\mathbf{V} = \mathbf{v} - \mathbf{u}$, and

$$\mathbf{j}_i = m_i \int d\mathbf{v}\, \mathbf{V}\, f_i \ ,$$

is the mass flux for species $i$ relative to the local flow

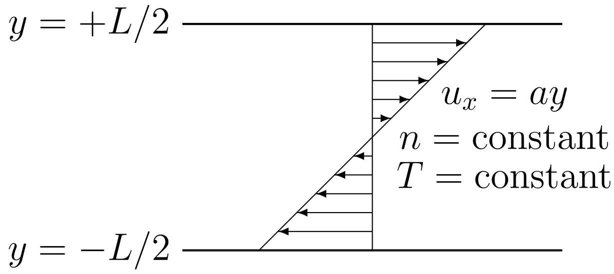
$$\mathbf{P} = \sum_i \mathbf{P}_i = \sum_i \int d\mathbf{v}\, m_i \mathbf{V}\mathbf{V}\, f_i \ ,$$

is the total pressure tensor,

$$\mathbf{q} = \sum_i \mathbf{q}_i = \sum_i \int d\mathbf{v}\, \tfrac{1}{2} m_i V^2 \mathbf{V}\, f_i \ ,$$

is the total heat flux.

## SIMPLE (UNIFORM) SHEAR FLOW




- Symmetry reasons: Mass and heat fluxes vanish. The pressure tensor is the only nonzero flux in the problem: rheological properties. Non-Newtonian behavior.

- Time evolution of the granular temperature $T$ arises from the balance of two opposite effects: viscous heating and dissipation in collisions.

$$\partial_t T = -\frac{2}{3n} a P_{xy} + (-\zeta T)$$

- A steady state is eventually reached in which the viscous heating is exactly balanced by collisional cooling effects$\rightarrow$ $T$ remains constant:

$$a P_{xy} = -\frac{3}{2}\zeta p, \quad p = nT$$

- We want to compute the momentum transport in this steady state.

- Simple shear flow becomes spatially uniform in the local Lagrangian frame moving with the flow velocity $\mathbf{u}=\mathbf{u}_i$: $f_i(\mathbf{r},\mathbf{v}) \to f_i(\mathbf{V})$.

$$-aV_y \frac{\partial}{\partial V_x} f_i = J_{ii}[f_i,f_i] + J_{ij}[f_i,f_j]$$

- We are interested in the elements of the partial pressure tensors $\mathbf{P}_i$. Taking velocity moments in the Boltzmann equation:

$$a\left(P_{1,\ell y}\delta_{kx} + P_{1,\ell x}\delta_{ky}\right) = A_{k\ell}^{11} + A_{k\ell}^{12}, \quad (1 \leftrightarrow 2)$$

$$A_{k\ell}^{ij} = m_i \int d\mathbf{V} V_k V_\ell J_{ij}[f_i,f_j]$$

- *Partial* pressures $p_i = n_i T_i = P_{i,kk}/3$. Previous studies assume the equality of the partial temperatures $T_i$: equipartition of granular energy between both species. However, this assumption is not well justified in general. This is one of the goals of our work.

- Closed problem once the cooling rate and the collisional moments $A_{k\ell}^{ij}$ are known. This requires the explicit knowledge of $f_i$: formidable task!!.

- We look for an approximate solution.

## APPROXIMATE SOLUTION

- Sonine polynomial expansion with a Gaussian measure. We retain only the first two terms:

$$f_i(\mathbf{V}) \to f_{i,M}(\mathbf{V}) \left[ 1 + \frac{m_i}{2T_i} \left( \frac{P_{i,k\ell}}{p_i} - \delta_{k\ell} \right) \left( V_k V_\ell - \frac{1}{3} V^2 \delta_{k\ell} \right) \right]$$

- Gaussian prefactors $f_{i,M}$ are defined at the temperature of species $i$, i.e.,

$$f_{i,M}(\mathbf{V}) = n_i \left( \frac{m_i}{2\pi T_i} \right)^{3/2} \exp\left( -\frac{m_i V^2}{2T_i} \right)$$

- New feature of our Sonine solution. The Maxwellians $f_{i,M}$ for the two species can be quite different due to the partial temperature differences.

- Dimensionless quantities. Let us introduce a characteristic collision frequency $\nu = \sqrt{\pi} n \sigma_{12}^2 v_0$, where $v_0 = \sqrt{2T(m_1+m_2)/m_1 m_2}$ is a thermal velocity:

$$T^* = \nu^2/a^2, \quad \mathbf{P}_i^* = \mathbf{P}_i/n_i T, \quad \mathbf{P}^* = (n_1/n)\mathbf{P}_1^* + (n_2/n)\mathbf{P}_2^*$$

- In the steady state $T^*$, $\gamma = T_1/T_2$, and $\mathbf{P}^*$ are *only* functions of

$$\{m_1/m_2, n_1/n_2, \sigma_1/\sigma_2, \alpha_{11}, \alpha_{22}, \alpha_{12} = \alpha_{21}\}$$

- To check the reliability of the Sonine approximation: comparison with Monte Carlo simulations (DSMC method).

## CONCLUSIONS

- Rheological properties of a low-density granular mixture subjected to a simple shear flow have been obtained by means of a first-Sonine polynomial expansion and the Direct Simulation Monte Carlo method. We are interested in the steady state where the effect of viscosity is compensated for by the dissipation in collisions. Our description applies for arbitrary values of the reduced shear rate $a$ and the inelasticity and not restriction on the values of the mechanical parameters are imposed.

- In contrast to previous studies, the theory predicts and the simulation confirms that the partial temperatures of each species are different. The extent of the equipartition violation depends on the degree of dissipation and the mechanical diferences of the particles.

- With respect to the rheological properties, theory and simulation presents a good quantitative agreement, especially for the shear stress $P_{xy}^*$. Kinetic theory also predicts normal stresses, although the discrepancies between theory and simulation are larger than those found for $T_1/T_2$ or $P_{xy}^*$.

- The effects of the two *different* partial temperatures on the rheological properties of the mixture are in general significant. This justifies the need of a *multi-temperature* theroy to capture the trends observed in the simulation.