IMPURITIES IN INELASTIC MAXWELL MODELS

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Granular fluids → Smooth hard spheres with inelastic collisions (IHS)

Difficulties to evaluate transport coefficients for IHS from the Boltzmann equation (BE), especially for multicomponent systems

Explicit results → Sonine polynomial expansion

For practical purposes only the leading terms are retained

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Hydrodynamics for a granular binary mixture at low density

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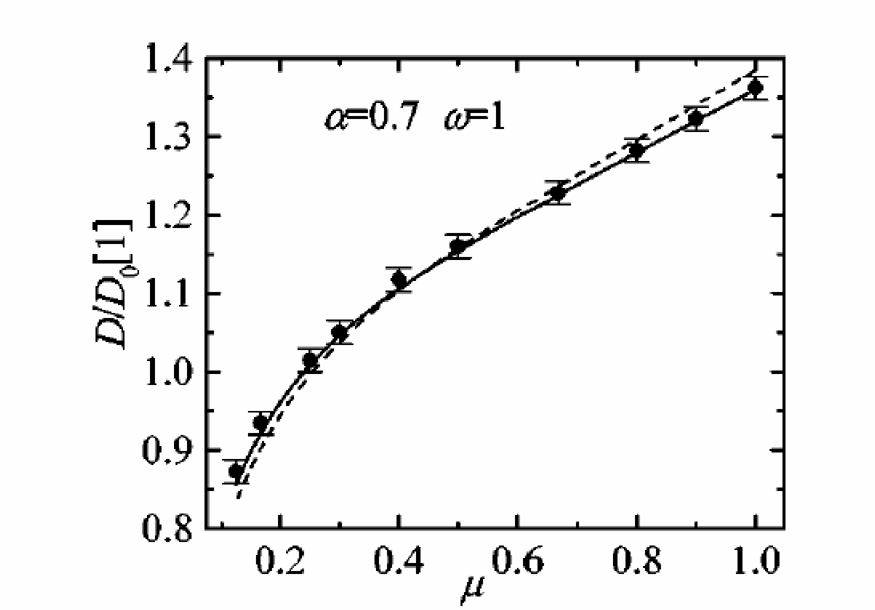

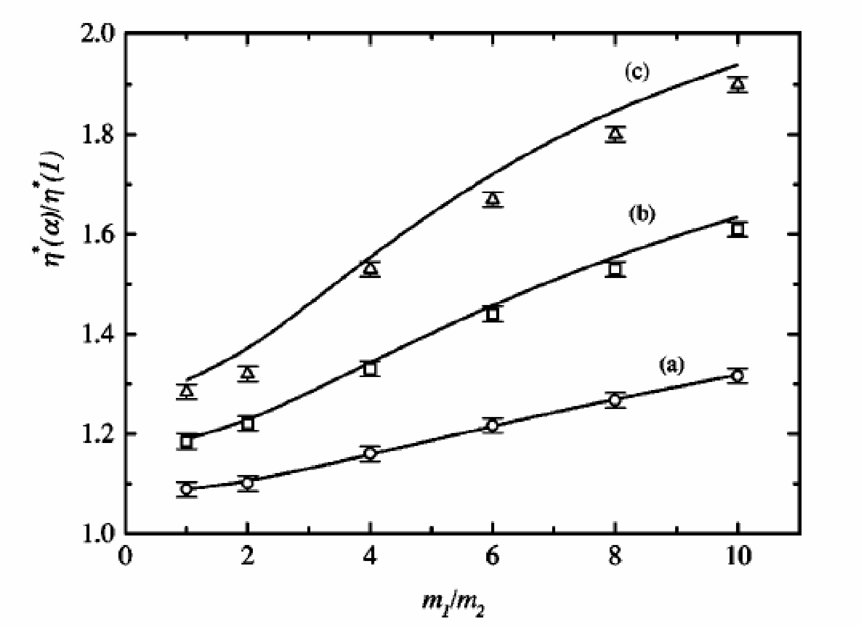

Alternative: Inelastic Maxwell models (IMM)

Its collision rate is velocity independent but their collision rules are the same as for IHS

Potential interaction ??

Perhaps, the cost of sacrificing physical realism can be in part compensated by getting **exact** results

IMM mainly used in homogeneous situations (overpopulated high energy tails)

Much less is known for **inhomogeneous states**

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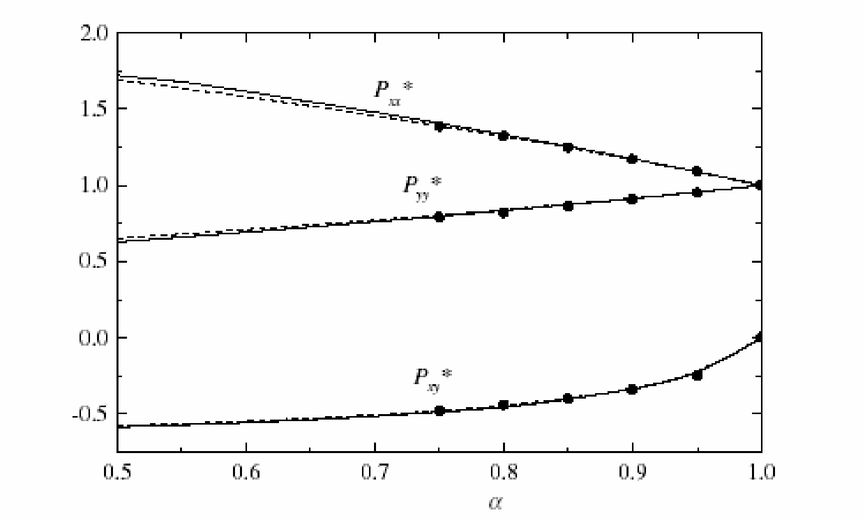
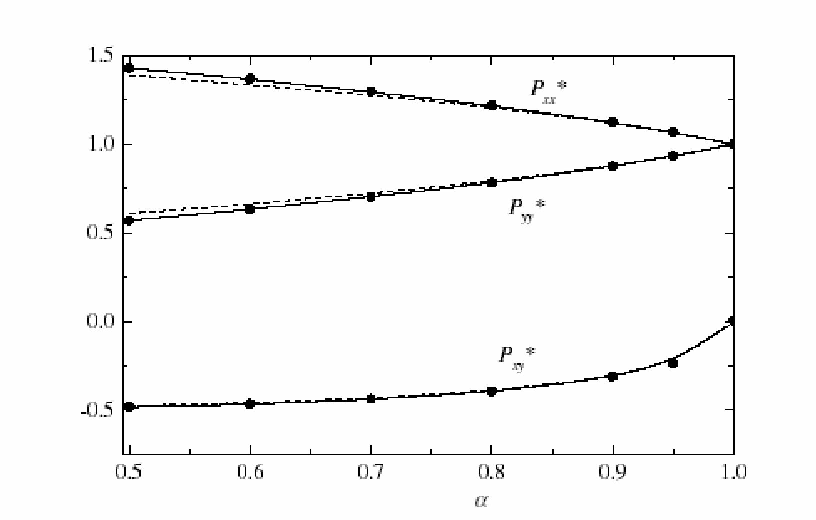
Nonlinear Transport in Inelastic Maxwell Mixtures Under Simple Shear Flow

Vicente Garzó[1]

Good agreement with IHS





OBJECTIVE: Transport properties of impurities immersed in a granular gas in HCS

Homogeneous cooling state with $T(t)$

$$\frac{1}{2}\zeta\frac{\partial}{\partial \mathbf{v}}\cdot(\mathbf{v}f)=J[\mathbf{v}|f,f]$$

$$\partial_t T=-\zeta T$$

$$J\left[\mathbf{v}_1|f,f\right]=\frac{\omega}{n\Omega_d}\int d\mathbf{v}_2\int d\widehat{\boldsymbol{\sigma}}\left[\alpha^{-1}f(\mathbf{v}_1',t)f(\mathbf{v}_2',t)-f(\mathbf{v}_1,t)f(\mathbf{v}_2,t)\right]$$

Free parameter

$$\zeta(\alpha)=\frac{1-\alpha^2}{2d}\omega$$

Coefficient of restitution

Some impurities are added
Transport generated by the presence of a (weak) concentration gradient

$$\partial_t f_0 + \mathbf{v} \cdot \nabla f_0 = J[\mathbf{v}|f_0, f]$$

$$J\left[\mathbf{v}_1|f_0, f\right] = \frac{\omega_0}{n_0 \Omega_d} \int d\mathbf{v}_2 \int d\widehat{\boldsymbol{\sigma}} \left[\alpha_0^{-1} f_0(\mathbf{r}, \mathbf{v}_1'; t) f(\mathbf{r}, \mathbf{v}_2'; t) - f_0(\mathbf{r}, \mathbf{v}_1; t) f(\mathbf{r}, \mathbf{v}_2; t)\right]$$

Impurity-gas collisions

$$\mathbf{v}_1' = \mathbf{v}_1 - \frac{m}{m + m_0}\left(1 + \alpha_0^{-1}\right)(\widehat{\boldsymbol{\sigma}} \cdot \mathbf{g}_{12})\widehat{\boldsymbol{\sigma}}, \quad \mathbf{v}_2' = \mathbf{v}_2 + \frac{m_0}{m + m_0}\left(1 + \alpha_0^{-1}\right)(\widehat{\boldsymbol{\sigma}} \cdot \mathbf{g}_{12})\widehat{\boldsymbol{\sigma}}$$

In the absence of diffusion, impurities are in HCS

$$\frac{1}{2}\zeta_0 \frac{\partial}{\partial \mathbf{v}} \cdot (\mathbf{v} f_0) = J[\mathbf{v}|f_0, f]$$

$$\partial_t T_0 = -\zeta_0 T_0$$

Cooling rate associated with

$$\frac{d}{2} n_0 T_0 = \int d\mathbf{v}\, \frac{m_0}{2} v^2 f_0(\mathbf{v})$$

$$\zeta_0 = \frac{2\omega_0}{d}\mu(1+\alpha_0)\left[1 - \frac{\mu}{2}(1+\alpha_0)(1+\theta)\right]$$

$$\mu = m/(m+m_0)$$

$$\theta = m_0 T / m T_0$$

Due to inelasticity $T \neq T_0$

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Breakdown of Energy Equipartition in a 2D Binary Vibrated Granular Gas

Klebert Feitosa* and Narayanan Menon†

$$(T/T_0)\partial_t(T_0/T) = \zeta - \zeta_0 \longrightarrow \zeta = \zeta_0$$

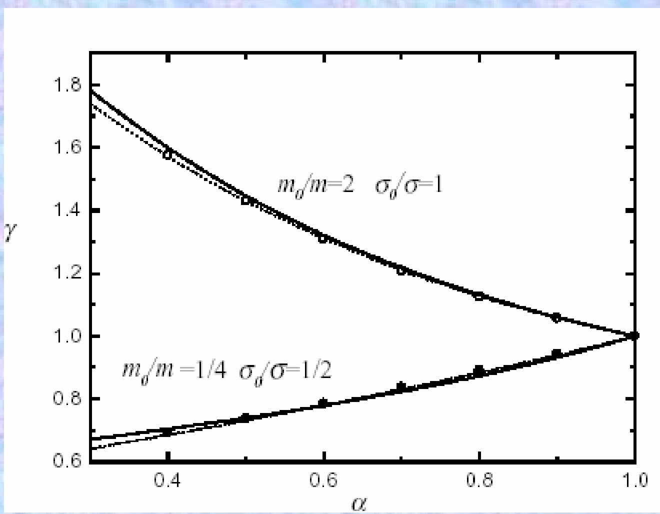
HCS condition

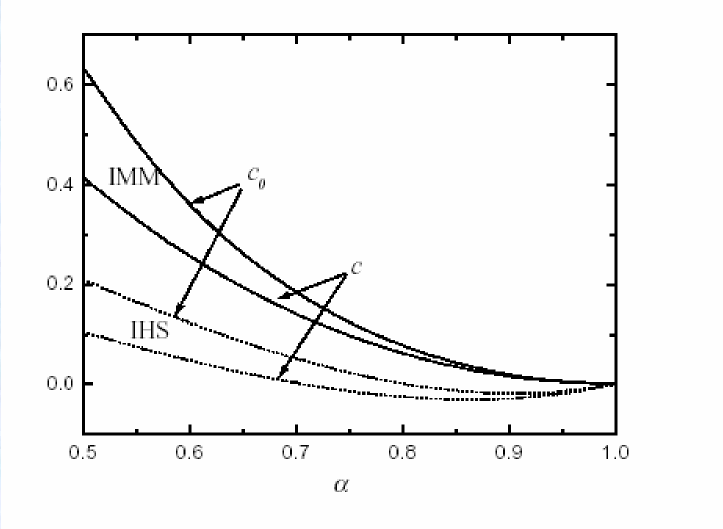
We adjust free paramaters of the model to get the same cooling rates as for IHS

$$\omega = \sqrt{\frac{2}{\pi}}\Omega_d \nu, \quad \omega_0 = \frac{\Omega_d}{\sqrt{\pi}}\left(\frac{\overline{\sigma}}{\sigma}\right)^{d-1}\left(\frac{1+\theta}{\theta}\right)^{1/2}\nu,$$

$$\overline{\sigma} = (\sigma + \sigma_0)/2$$

$$\nu = n\sigma^{d-1}\sqrt{2T/m}$$





Transport properties of impurities

$$f_0 = f_0^{(0)} + \epsilon f_0^{(1)} + \cdots$$

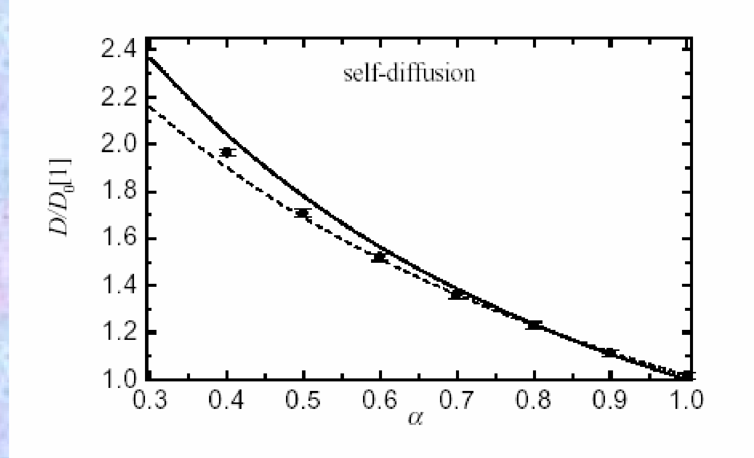
Chapman-Enskog expansion

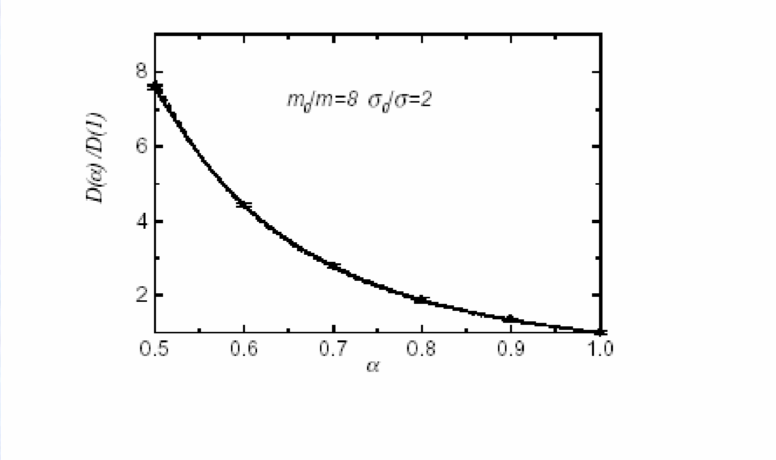
$$\mathbf{j}_0^{(1)} = \int d\mathbf{v}\, m_0 \mathbf{v} f_0^{(1)} = -m_0 D \nabla x_0$$

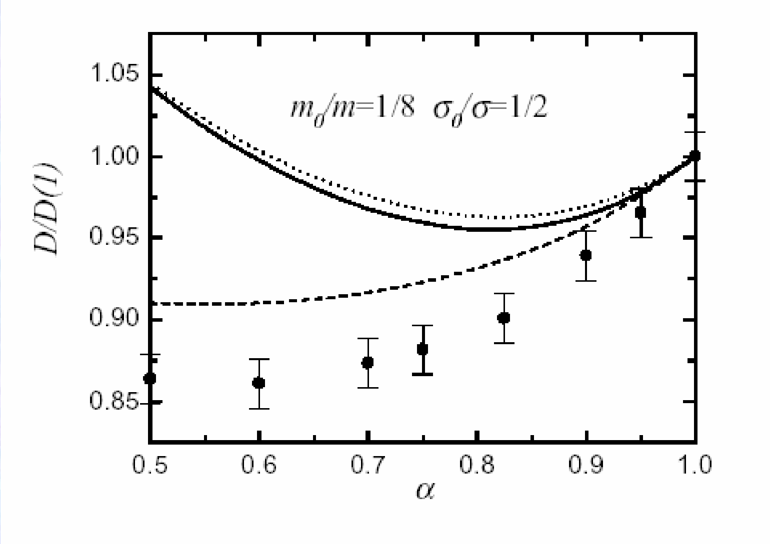
Diffusion coefficient

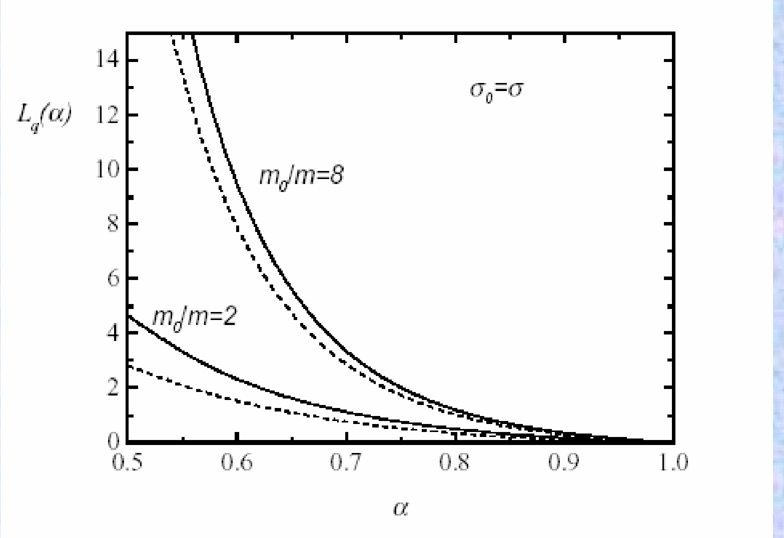
$$\mathbf{J}_q^{(1)} = \int d\mathbf{v} \left(\frac{m_0}{2} v^2 - \frac{d+2}{2} T \right) \mathbf{v} f_0^{(1)} = -\frac{nT^2}{m_0 \nu} L_q \nabla x_0$$

Dufour coefficient


self-diffusion
$D/D_0[1]$
α


$m_0/m=8$ $\sigma_0/\sigma=2$
$D(\alpha)/D(1)$
α


$m_0/m=1/8$ $\sigma_0/\sigma=1/2$
$D/D(1)$
α


$\sigma_0=\sigma$
$L_q(\alpha)$
$m_0/m=8$
$m_0/m=2$
α

CONCLUSIONS

1. IMM capture the main trends observed for IHS, at least for not too large dissipation.

2. Good agreement for diffusion coefficient but discrepancies between both models increase for higher-degree velocity moments (fourth-cumulants, Dufour coefficient,...)

3. The reliability of IMM as a model of granular fluids must be taken with caution

Thanks for your attention