Incorporation of Fluid Phase into Hydrodynamic Description of Granular Flows

Christine Hrenya
Vicente Garzó
Shankar Subramaniam
Rahul Garg
Sudheer Tennetti

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Aside: “Local” Particle Transport Mysteries

~400-800 AD: settled by Polynesian

1600’s: European “discovery’

…*1000 years of isolation*

~2600 miles from Tahiti

~2500 miles from Santiago (6 hour fl

Easter Island’s longest dimension: 16

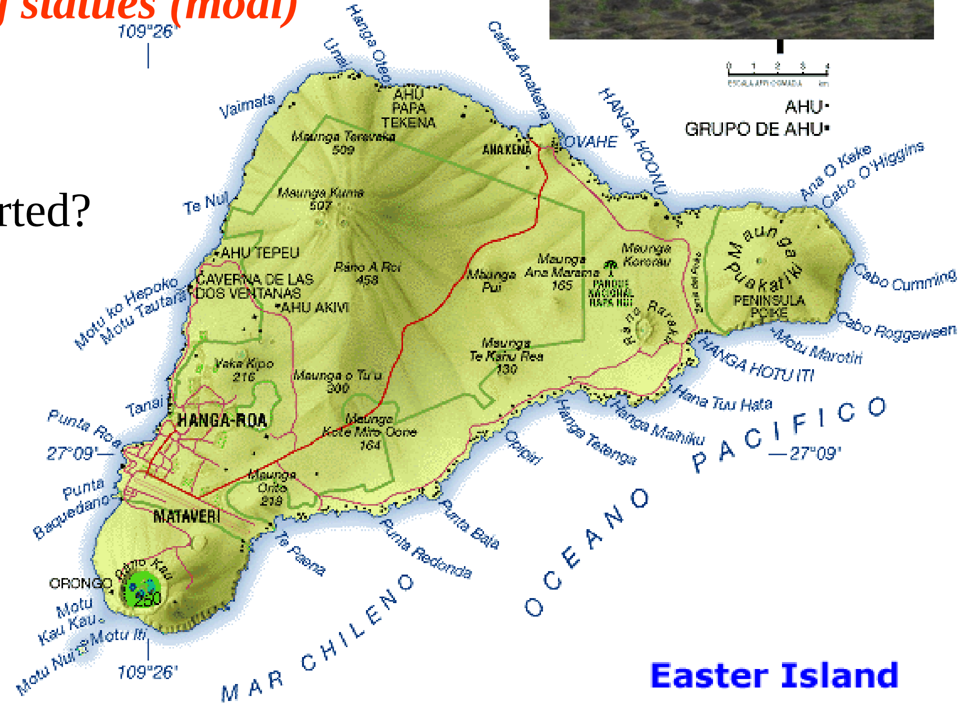
…*Most remote, inhabited place on E*

Inhabitants put all their efforts into building statues (moai)

~6-30 feet tall (largest erected ~100 ton)

~900 total catalogued (~200 erect)

Mystery: how were these “particles” transported?

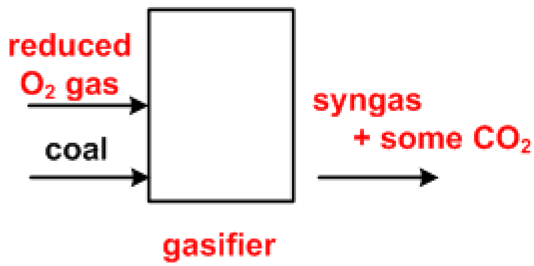


Aside: "Local" Particle Transport Mysteries

Modern-day researchers conducting field work in the prediction of avalanches during hike in Andes (SWGM, circa 2006)

Motivation

Energy Production via Gasification

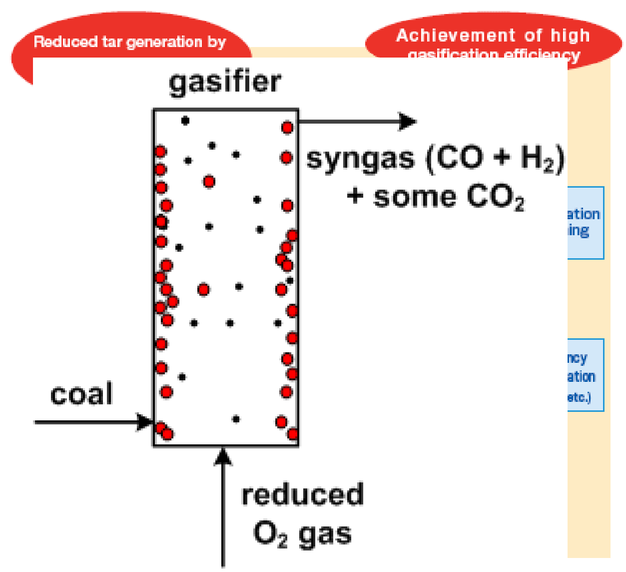


Advantages

- *60% theoretical efficiency*
- *smaller amounts of CO_2*
- *CO_2 is easily sequestered*
- *applicable to renewables like biomass; short-term (<250yrs) & long-term implications*

Where do we fit in?

- Gasifiers are *multiphase* reactors : gas + solids
- The solid (coal) particles display *segregation & clustering*
- Better gas-solid contacting (improved mixing) is expected to lead to increased efficiencies



Modeling of Gas-solid Flows

Continuum ("Two-fluid") Description

- Gas phase: Navier-Stokes + turbulence + drag force
- Solids phase: Kinetic-theory-based models + fluid-phase interactions

Overall Goal: Extension to polydisperse systems

- Colorado: solid-phase hydrodynamics + lab-scale experiments
- Iowa State: drag force + turbulence + population balances
- Princeton: drag force
- PSRI: pilot-scale experiments

Current Objective: ***Incorporation of gas-phase effects into kinetic-theory-based models for solid phase***

Physical Picture

Recall fluid-solid interaction force (drag force)

$$F_{fluid} = F_n + F_t = \int_0^{2\pi}\int_0^{\pi} \left(-p\big|_{r=R} \cos\theta \right) R^2 \sin\theta \, d\theta d\phi + \int_0^{2\pi}\int_0^{\pi} \left(\tau_{r\theta}\big|_{r=R} \sin\theta \right) R^2 \sin\theta \, d\theta d\phi$$

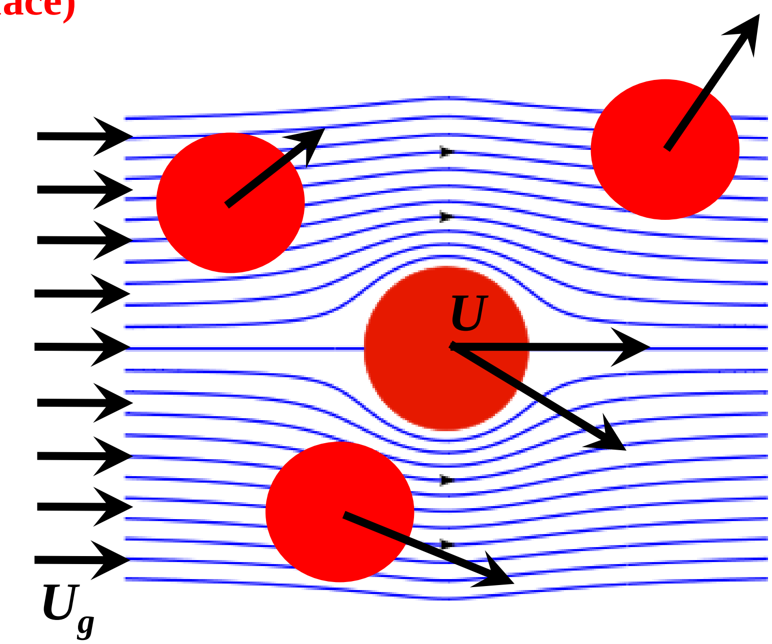
= *f* (fluid velocity at surface)

***Mean* fluid force on *single* particle**

$$F_{fluid} = f\left(U_g - U\right)$$

Velocity & pressure fields (& thus fluid force) change with:

- Fluctuations in particle velocity
- Fluctuations in gas velocity

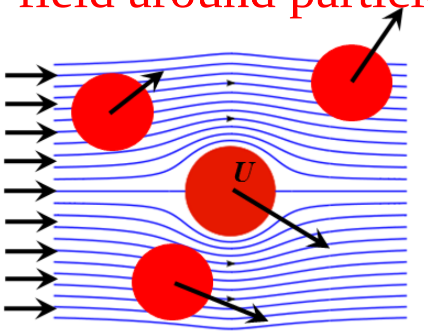


➡ ***Mean* vs. *Instantaneous* Fluid Force**

Incorporation of Instantaneous Fluid Force

Alternative 1: DEM (solids) /DNS (fluid) – resolve flow field around particles

+ fluid force is "output"
- too computationally expensive (no-slip BC at each moving particle surface)



Alternative 2: Two-fluid model – "averaged" flow field over several particles

+ computationally feasible (single equation of motion for each phase)
- fluid effects are "input" – model is needed to subsume *instantaneous* effects

Q1: Impact on governing equations (additional terms)?

Q2: Impact on constitutive relations for solid phase ($\mathbf{P}$, $\mathbf{q}$, ζ)?

Current Approach

(i) use DEM/DNS simulations to develop model for instantaneous force

(ii) incorporate this force model into starting kinetic equation & derive hydrodynamic description

More details…

Basic Idea: Incorporation of fluid force into Enskog kinetic equation

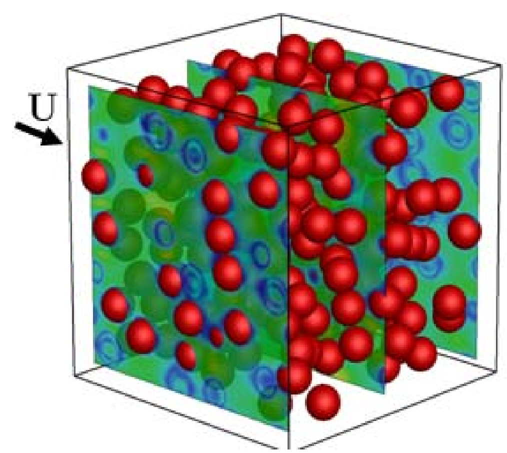
transient *convective* *gravity* *collisions*

$$\frac{\partial}{\partial t} f + v_i \frac{\partial f}{\partial x_i} + \boxed{\frac{\partial}{\partial v_i}\left(\frac{F_{fluid,i}}{m}\right)} + g_i \frac{\partial}{\partial v_i} f = J$$

***instantaneous** fluid force*
on single particle

DEM/DNS technique for closure: IBM (Immersed Boundary Method) based model of $F_{fluid,i}$ as function of:

- Hydrodynamic variables: ϕ, U_i, T, U_{gi}
- Physical parameters: m, d, α, μ_g , ρ_g



IBM-based model for acceleration

coefficients extracted from IBM simulations

$$\frac{F_{fluid,i}}{m} = A_i = -\frac{\beta_{IBM}}{m}\left(U_i - U_{gi}\right) - \frac{1}{m}\gamma_{ij}V_j - B_{ij}dW_j$$

instantaneous particle acceleration — *mean particle velocity* — *mean gas velocity* — *fluctuating particle velocity* — *Wiener process increment (stochastic model for fluctuating fluid velocity)*

Use IBM simulations to find β^*, γ_{ij}^*, and B_{ij}^* as functions of

- ϕ — solids volume fraction
- ρ_s/ρ_f — density ratio
- $Re_m = \dfrac{(1-\phi)\rho_g d\left|\mathbf{U} - \mathbf{U_g}\right|}{\mu_g}$ — *Re* based on mean flow
- $Re_T = \dfrac{\rho_g d}{\mu_g}\sqrt{\dfrac{T}{m}}$ — *Re* based on particle velocity fluctuations

Resulting Hydrodynamic Description

Balance Equations (Solid-Phase Momentum & Granular Energy)

$$D_t\mathbf{U} + \frac{1}{mn}\nabla\mathbf{P} = \boxed{-\frac{\beta_{IBM}}{m}\left(\mathbf{U}-\mathbf{U}_g\right)} + \mathbf{g}$$

mean drag

$$D_tT + \frac{2}{3n}\left(\nabla\cdot\mathbf{q} + P_{ij}\nabla_jU_i\right) = -\zeta T \boxed{-\frac{2}{3\rho}\gamma_{ij}P_{ij}^{k}} \boxed{+\frac{\rho}{3n}B_{ij}B_{ij}}$$

sink due to viscous drag

source due to fluid-particle fluctuations

Explicit Constitutive relations obtained for ζ, P, and q:

- Cooling rate $\zeta^{(0)} = \zeta^{(0)}\left(B_{ij}\right)$
- Cooling rate TC $\zeta_U = \zeta_U\left(\gamma_{ij}, B_{ij}\right)$
- Shear viscosity $\eta = \eta\left(\gamma_{ij}, B_{ij}\right)$
- Bulk viscosity $\lambda = \lambda\left(B_{ij}\right)$
- Conductivity $\kappa = \kappa\left(\gamma_{ij}, B_{ij}\right)$
- Dufour coefficient $\mu = \mu\left(\gamma_{ij}, B_{ij}\right)$

Base Case: Massive Particles (St>>1) and Stokes flow (Re_m<<1)

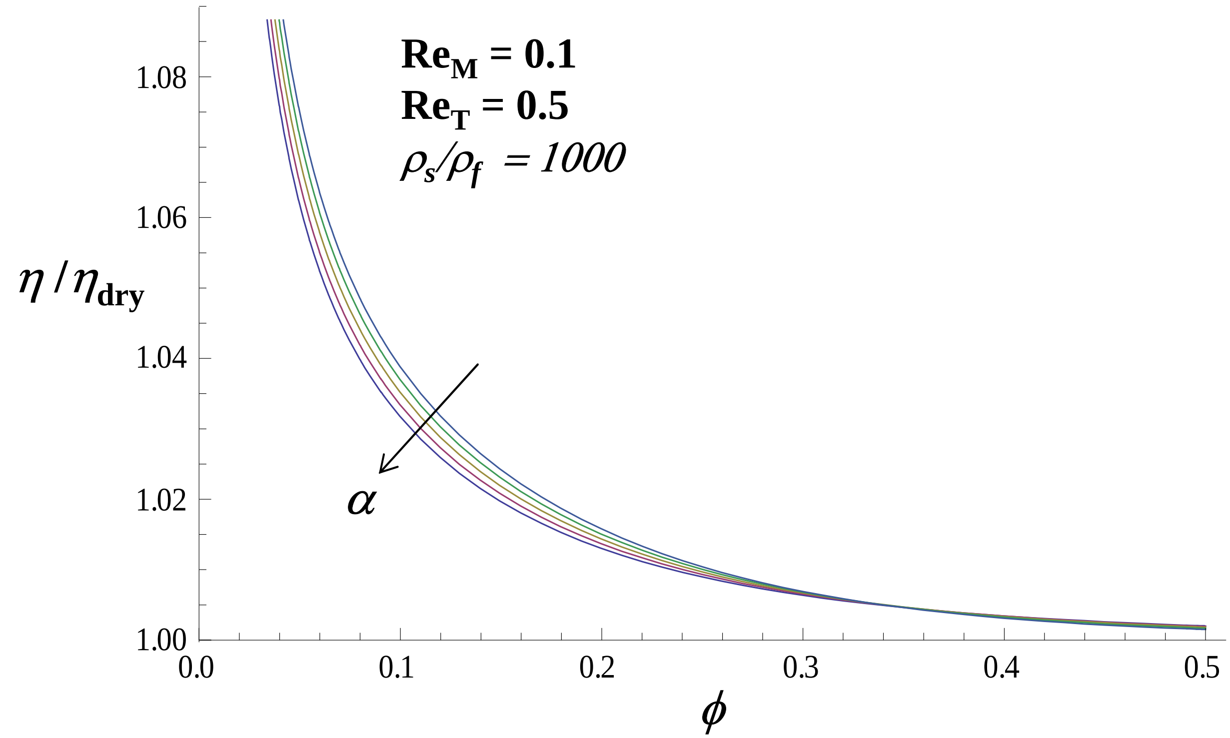
Typical Ranges in CFB (Circulating Fluidized Bed) riser

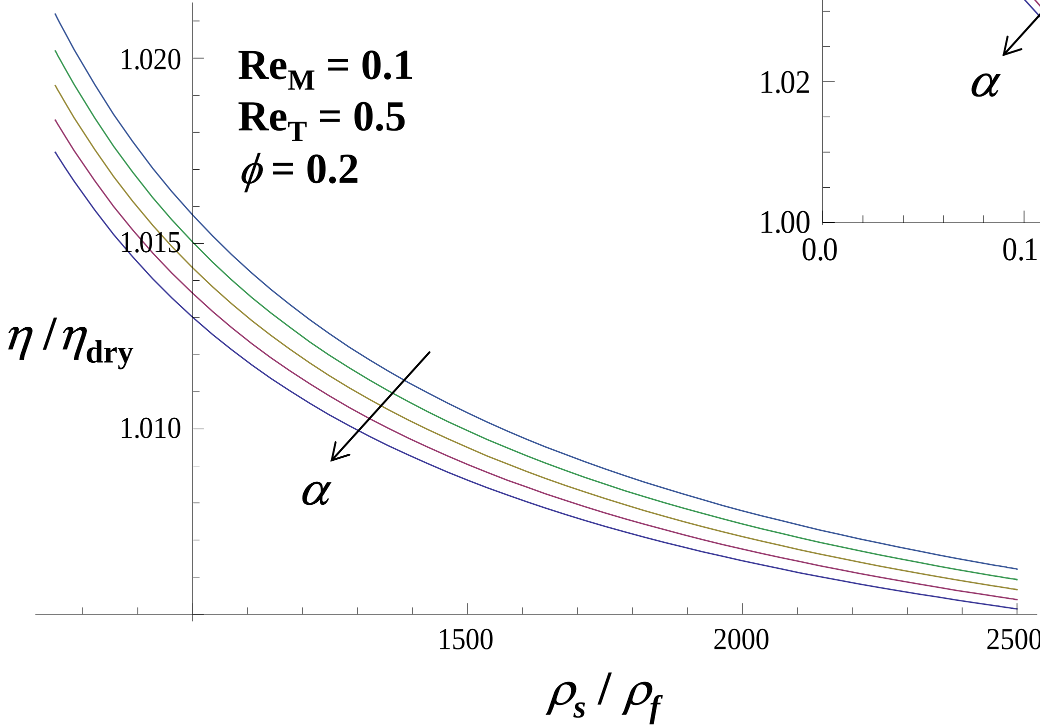
- ϕ 0.01 – 0.5
- ρ_s/ρ_f 800 – 250 ⟶ *high St*
- Re_m 0.1 – 50 ⟶ *low – moderate Re* ➡ *only low Re_m = 0.1 – 1 considered here*
- Re_T 0.5 – 5

Summary of Results

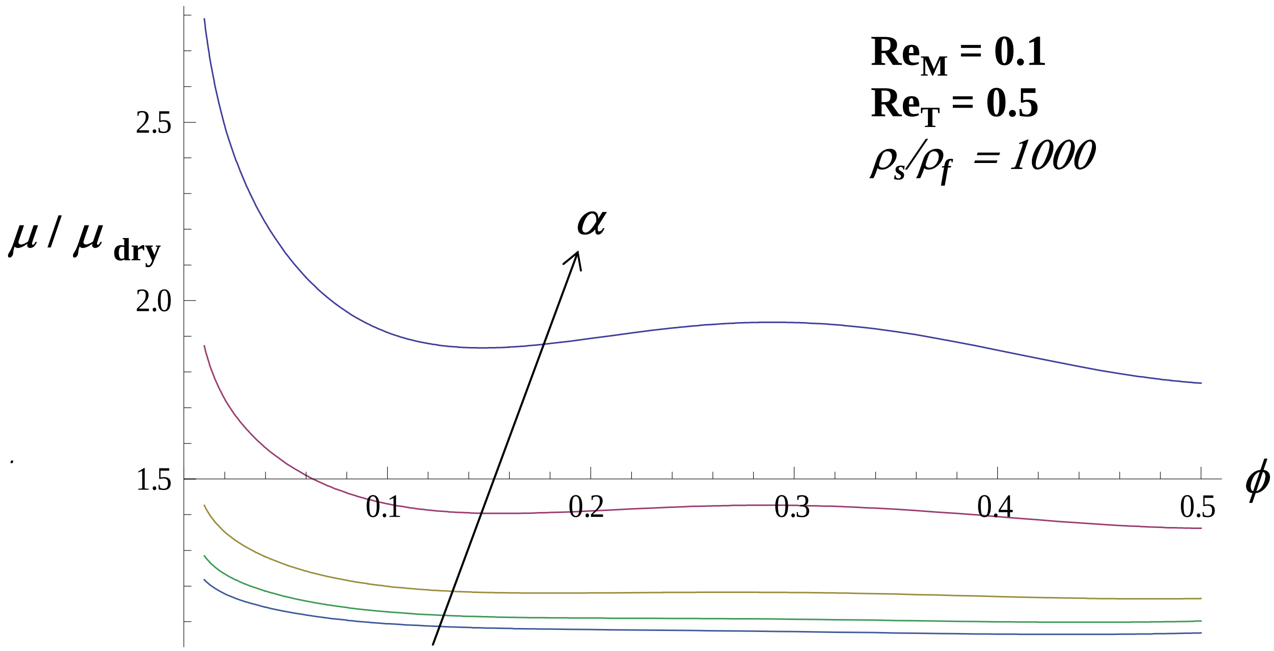
- *Negligible* gas-phase influence
  - Cooling rate ($\zeta_{(0)}$, ζ_U)
  - Bulk viscosity (λ)
  - Conductivity (κ)
- *Non-negligible* gas-phase influence
  - Shear viscosity (η)
  - Dufour coefficient (μ)

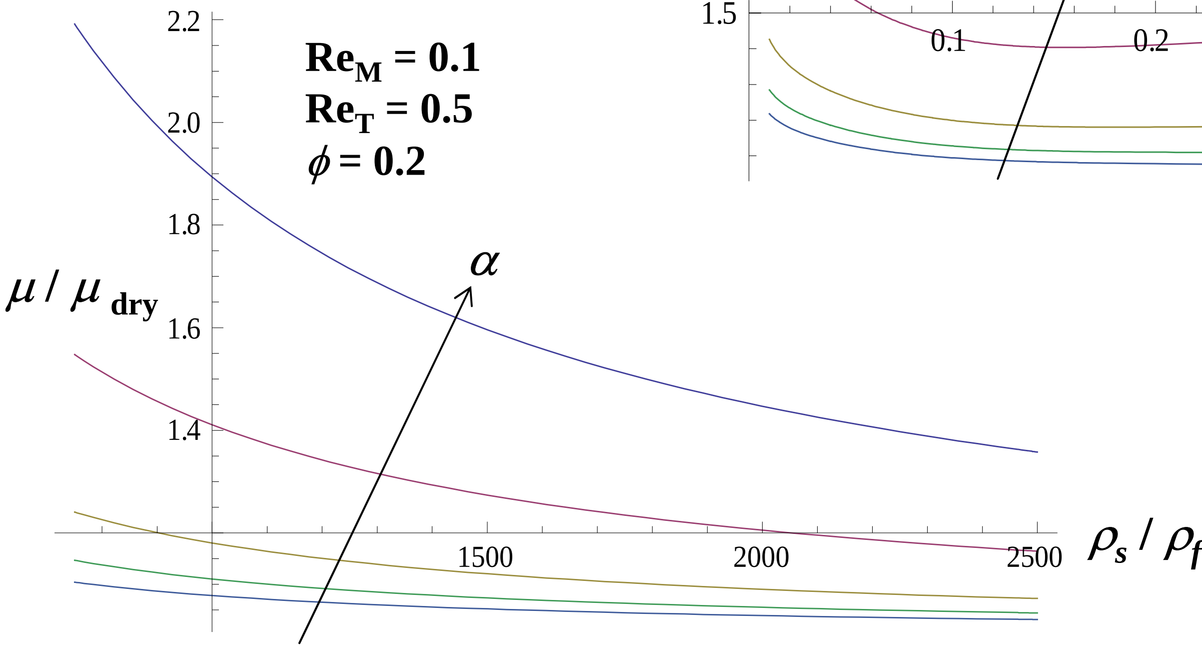
Shear Viscosity





Dufour Coefficient





Summary

IBM-based model for instantaneous fluid acceleration has been incorporated into Enskog equation, and corresponding hydrodynamic description derived

- Additional source/sink in momentum and granular energy balances
- Modification of constitutive closures
  - *For limiting case of Re_m << 1 and St>>1:* non-negligible gas-phase influence on shear viscosity and Dufour coefficient
- Framework extendible to non-limiting cases once IBM coefficients are extracted (coming soon…)