FLOW INSTABILITIES IN UNDRIVEN GRANULAR FLUIDS AT MODERATE DENSITIES

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INTRODUCTION

Granular systems are systems constituted by macroscopic grains: Sand, rice, sugar, snow, pills,…….Ubiquitous in our daily lives

In some cases, they can behave as a solid. In others (external excitation), granular medium is more similar to a gas.

Granular gas: gas constituted by *smooth* hard spheres with *inelastic* collisions

Real granular systems characterized by some degrees of *polidispersity* in density and size: *Multicomponent* granular systems

Mechanical parameters: $\{m_i, \sigma_i, \alpha_{ij}\}, \quad i = 1, \cdots, s$

Direct collision:

$$\mathbf{v}_1^* = \mathbf{v}_1 - \frac{m_j}{m_i + m_j}\left(1 + \alpha_{ij}\right)(\widehat{\boldsymbol{\sigma}} \cdot \mathbf{V}_{12})\widehat{\boldsymbol{\sigma}}$$

$$\mathbf{v}_2^* = \mathbf{v}_2 + \frac{m_i}{m_i + m_j}\left(1 + \alpha_{ij}\right)(\widehat{\boldsymbol{\sigma}} \cdot \mathbf{V}_{12})\widehat{\boldsymbol{\sigma}}$$

$$(\Delta E)_{ij} = \frac{1}{2}\left(m_i v_1^{*2} + m_j v_2^{*2} - m_i v_1^2 - m_j v_2^2\right) = -\frac{1}{2}\frac{m_i m_j}{m_i + m_j}(1 - \alpha_{ij}^2)(\mathbf{V}_{12} \cdot \widehat{\boldsymbol{\sigma}})^2$$

REVISED ENSKOG KINETIC THEORY

S-multicomponent mixture of smooth hard spheres or disks of masses m_i, diameters σ_i, and coefficients of restitution α_{ij}

At a kinetic level: $f_i(\mathbf{r}_1,\mathbf{v}_1;t)$

$$\left(\partial_t + \mathbf{v}_1 \cdot \nabla_{\mathbf{r}_1} + m_i^{-1}\mathbf{F}_i(\mathbf{r}_1) \cdot \nabla_{\mathbf{v}_1}\right) f_i(\mathbf{r}_1, \mathbf{v}_1; t) = C_i(\mathbf{r}_1, \mathbf{v}_1; t)$$

$$C_i(\mathbf{r}_1, \mathbf{v}_1; t) = \sum_{j=1}^{s} \sigma_{ij}^{d-1} \int d\mathbf{v}_2 \int d\widehat{\boldsymbol{\sigma}} \Theta(\widehat{\boldsymbol{\sigma}} \cdot \mathbf{g}_{12})(\widehat{\boldsymbol{\sigma}} \cdot \mathbf{g}_{12}) \times \left(\alpha_{ij}^{-2} f_{ij}(\mathbf{r}_1, \mathbf{v}_1'', \mathbf{r}_1 - \boldsymbol{\sigma}_{ij}, \mathbf{v}_2''; t) - f_{ij}(\mathbf{r}_1, \mathbf{v}_1, \mathbf{r}_1 + \boldsymbol{\sigma}_{ij}, \mathbf{v}_2; t)\right)$$

Two-particle distribution function

$$f_{ij} \to \chi_{ij} f_i f_j$$

Balance equation for the partial densities

$$D_t n_i + n_i \nabla \cdot \mathbf{U} + m_i^{-1} \nabla \cdot \mathbf{j}_i = 0$$

Balance equation for the flow velocity

$$\rho D_t U_\beta + \nabla_\gamma P_{\gamma\beta} = \sum_{i=1}^{s} n_i(\mathbf{r}, t) F_{i\beta}(\mathbf{r})$$

Balance equation for the granular temperature

$$\frac{d}{2} n (D_t + \zeta) T + P_{\gamma\beta} \nabla_\gamma U_\beta + \nabla \cdot \mathbf{q} - \frac{d}{2} T \sum_{i=1}^{s} \frac{\nabla \cdot \mathbf{j}_i}{m_i} = \sum_{i=1}^{s} \frac{\mathbf{F}_i \cdot \mathbf{j}_i}{m_i}$$

Cooling rate

$$D_t \equiv \partial_t + \mathbf{v} \cdot \nabla$$

Constitutive equations (binary mixture)

$$\mathbf{j}_1 = -\frac{m_1^2 n}{\rho} D_{11} \nabla \ln n_1 - \frac{m_1 m_2 n_2}{\rho} D_{12} \nabla \ln n_2 - \rho D_T \nabla \ln T$$

$$P_{\gamma\beta} = p\delta_{\gamma\beta} - \eta \left(\nabla_\gamma U_\beta + \nabla_\beta U_\gamma - \frac{2}{d} \nabla \cdot \mathbf{U} \right) - \kappa \nabla \cdot \mathbf{U}$$

$$\mathbf{q} = -T^2 D_{q,1} \nabla \ln n_1 - T^2 D_{q,2} \nabla \ln n_2 - \lambda \nabla T$$

Eight transport coefficients: $\left\{ D_{11}, D_{12}, D_T, \eta, \kappa, D_{q,1}, D_{q,2}, \lambda \right\}$

Parameter space: $\{m_1/m_2, \sigma_1/\sigma_2, x_1, \phi, \alpha_{11}, \alpha_{22}, \alpha_{12}\}$

INSTABILITIES IN FREELY COOLING *DENSE* GRANULAR BINARY MIXTURES

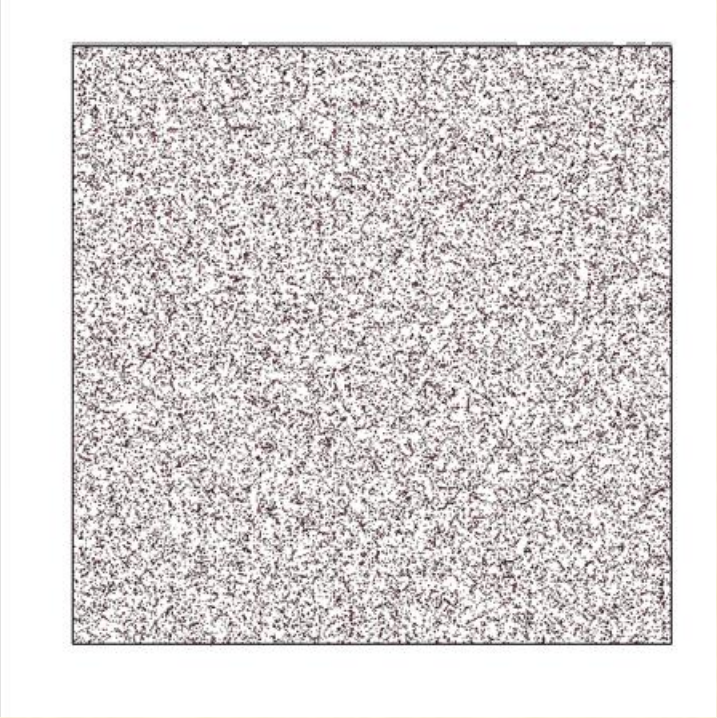
In contrast to ordinary fluids, *instabilities* (such as dynamic particle clusters) occur in the homogeneous cooling state (HCS) of granular gases

Pionnering work of Goldhirsch&Zanetti (PRL **70**, 1619 (1993))

Typical configuration of particles exhibing clustering

$$\alpha = 0.6, \phi = 0.05$$

(Peter Mitrano, CU)



General trends: (i) instabilities are more likely in larger domains; and (ii) velocity vortices manifest more readily than particle clusters

This is also a very ***clean*** problem to assess Navier-Stokes hydrodynamics derived from kinetic theory

For given values of the mechanical parameters of the system, there exists a **critical** system lenght demarcates (stable) homogeneous flow from one with *velocity-vortex* instabilities or one exhibiting the *clustering* instability

LINEAR STABILITY ANALYSIS

If the system length L>Lc, then the system becomes unstable

In most of the studied cases, the linear stability analysis predicts that the instability is driven by velocity vortices

$$L_c^* = 2\pi\sqrt{\frac{\eta^*}{2\zeta^*}}$$

Stringent *assessment* of kinetic theory calculations!!!

MONOCOMPONENT GRANULAR FLUIDS

VG, PRE **72**, 021106 (2005)

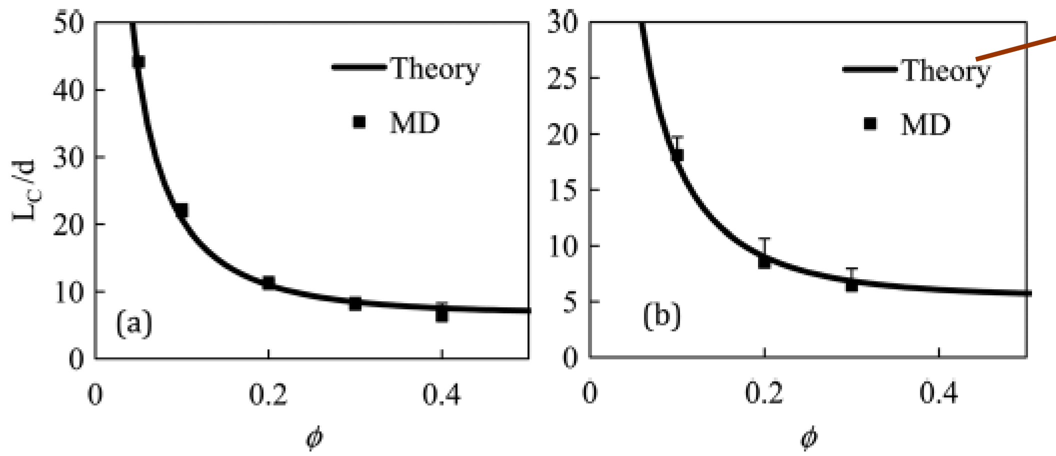


FIG. 8. Comparison of the critical length scale (L_C/d) for (a) $e=0.8$ and (b) $e=0.7$. The "error bars" correspond to the transition length scale range obtained from MD simulations (see Sec. IV A and Figure 5). Theoretical predictions are obtained from the linear stability analysis of Garzó (Ref. 23).

Mitrano, Dhal, Cromer, Pacella , Hrenya, Phys. Fluids **23**, 093303 (2011)

Standard Sonine approximation

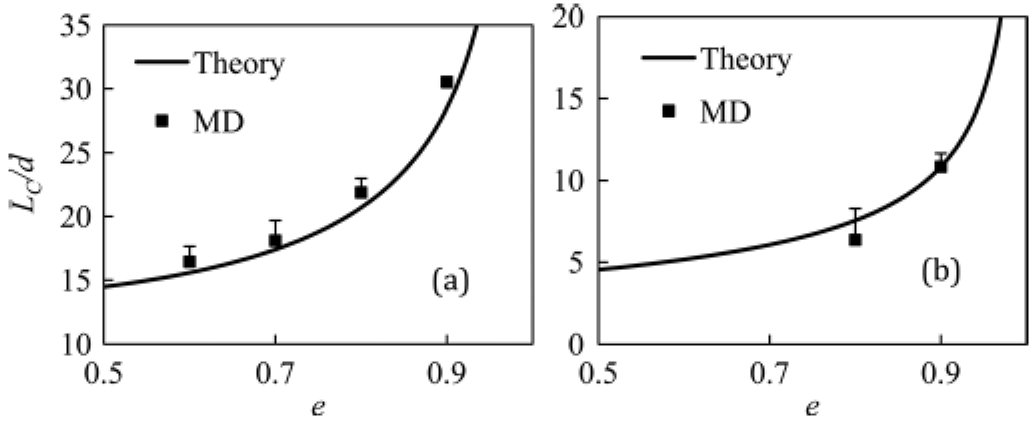


FIG. 9. Comparison of the critical length scale for (a) $\phi = 0.1$ and (b) $\phi = 0.4$. The "error bars" correspond to the transition length scale range obtained from MD simulations (see Sec. IV A and Figure 5). Theoretical predictions are obtained from a linear stability analysis of Garzó (Ref. 23).

HIGHLY DISSIPATIVE GRANULAR FLUIDS

Modified Sonine approximation (VG, Santos, Montanero, Physica A **376** 94 (2007))

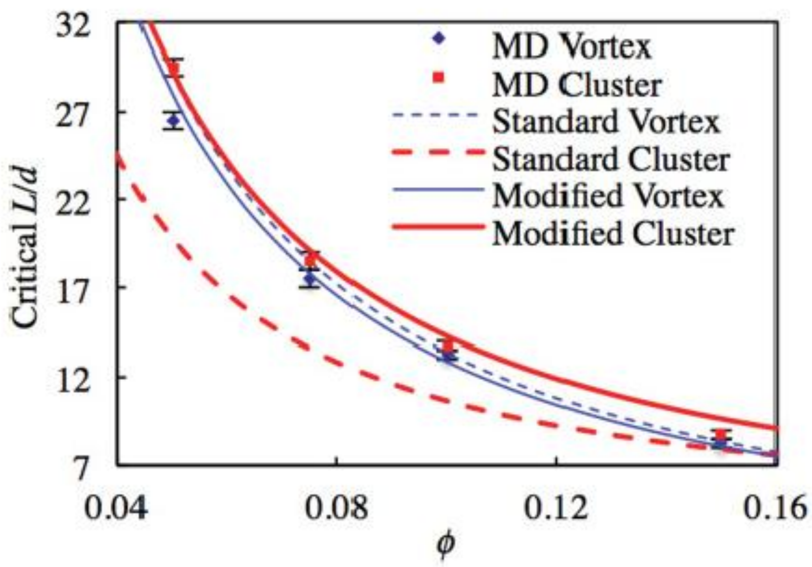


FIG. 4. (Color online) Critical length scale for vortex and cluster instabilities (i.e., L_{Vortex}/d and L_{Cluster}/d, respectively) plotted as a function of solids fraction for $e = 0.25$. The solid and dashed lines correspond to the modified and standard theories, respectively.

Mitrano, VG, Hilger, Ewasko, Hrenya, PRE **85**, 041303 (2012)

BINARY GRANULAR DENSE MIXTURES

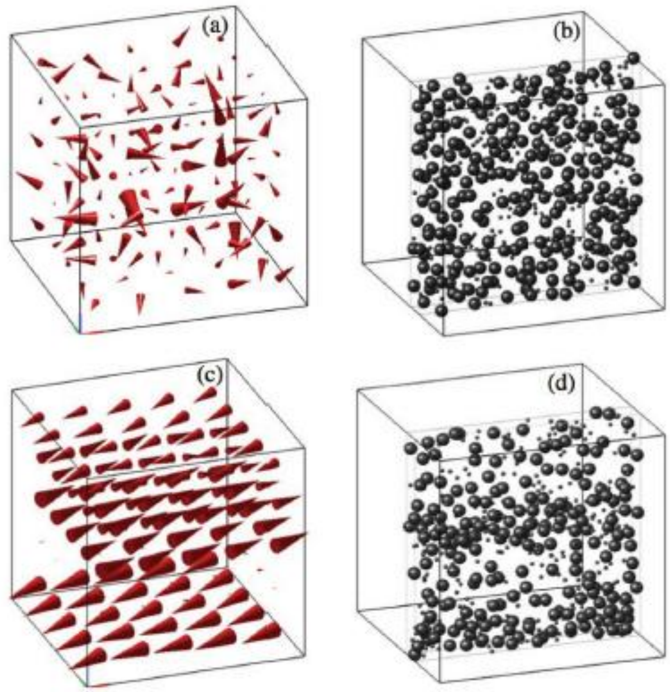


FIG. 1. (Color online) Visualizations from a MD simulation of an equimolar mixture ($x_1 = 0.5$) with $m_1/m_2 = 2$, $\sigma_1/\sigma_2 = 3$, $\phi = 0.2$, and $\alpha = 0.7$ of (a) stable, coarse-grained velocity field at five collisions per particle (or "cpp"), (b) stable particle positions at five cpp, (c) unstable, coarse-grained velocity field at 400 cpp, and (d) cluster systems at 400 cpp. A cell size of $L/5$ is used for local velocity averaging.

P. Mitrano, VG, C. Hrenya
PRE **89**, 0200201 (R) (2014)

BINARY GRANULAR DENSE MIXTURES

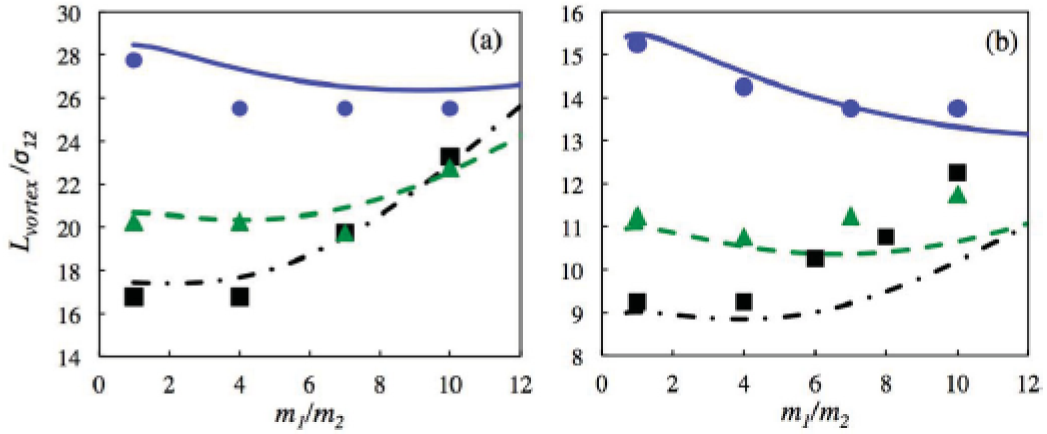


FIG. 2. (Color online) Critical length scale for velocity vortices as a function of the mass ratio m_1/m_2 with $x_1 = 0.1$, $\sigma_1/\sigma_2 = 1$ for (a) $\phi = 0.1$ and (b) $\phi = 0.2$. The data points correspond to MD simulations, while the lines are the theoretical predictions given by Eq. (3). (Blue) circles/solid line, (red) triangles/dashed line, and (black) squares/dot-dashed line correspond to $\alpha = 0.9$, $\alpha = 0.8$, and $\alpha = 0.7$, respectively. Error ranges are the size of the data points and are omitted.

BINARY GRANULAR DENSE MIXTURES

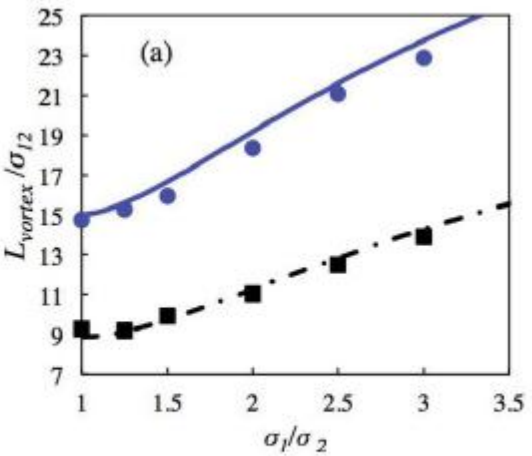


FIG. 3. (Color online) Critical length scale for velocity vortices as a function of (a) the ratio of diameters σ_1/σ_2 with $m_1/m_2 = 2$, $x_1 = 0.5$, and $\phi = 0.2$ and (b) the mole fraction x_1 with $m_1/m_2 = 6$, $\sigma_1/\sigma_2 = 1$, and $\phi = 0.2$. The meaning of symbols and lines is the same as that of Fig. 2.

CONCLUSIONS

✓Hydrodynamic description (derived from kinetic theory) appears to be a powerful tool for analysis and predictions of rapid flow gas dynamics of granular mixtures at ***moderate*** densities.

✓Instability of HCS: Good agreement with MD even for finite dissipation and moderate densities. Stringent test of kinetic theory results

Quantitative agreement could be improved at disparate masses by considering the second Sonine approximation to the ***shear viscosity***

Thanks for your attention