SEGREGATION IN MODERATELY DENSE GRANULAR BINARY MIXTURES

Vicente Garzó

Departamento de Física, Universidad de Extremadura, Badajoz, Spain

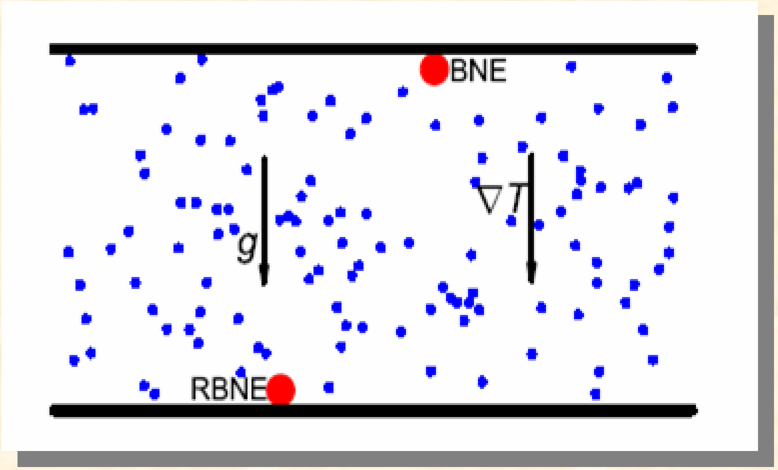
XVth International Congress on Rheology, August 3-8, 2008 Monterey (CA)

Segregation and **mixing** of dissimilar grains is one of the most interesting problems in vibrated granular mixtures

Several mechanisms have been proposed:
Archimedean buoyancy, void filling, convection, frictional properties,…..
thermal (Soret) diffusion

Large shaking amplitude ⟶ Sample of grains resembles a *granular gas*

Objective: Segregation problem driven by the presence of a thermal gradient and the gravitational field in a *moderately* dense ganular fluid



Model system: dense *granular* fluid+**intruder**

Mechanical parameters $\{m, m_0, \sigma, \sigma_0, \alpha, \alpha_0\}$

We assume that $\sigma_0 > \sigma$

Collisional dissipation

Experimental conditions: *inhomogeneous* steady state without convection (*zero* mass flux) and gradients along the z direction

$$-\Lambda \partial_z \ln T = \partial_z \ln(n_0/n)$$

$\Lambda > 0 \longrightarrow$ Intruder rises with respect fluid (**BNE**)

$\Lambda < 0 \longrightarrow$ Intruder falls with respect fluid (**RBNE**)

Early attempts:

- *Elastic* systems: Jenkins and Yoon PRL 2002
- *Quasielastic* particles: Trujillo, Alam & Herrmann EPL 2003
- *Dilute* gases: Brey et al. PRL 2005; Garzó EPL 2006

Our theory (i) goes beyond weak dissipation limit, (ii) combined effect of gravity and thermal gradient , (iii) applies for moderate densities

Hydrodynamic description to evaluate thermal diffusion Λ

a) *Momentum* balance equation:

$$\frac{\partial p}{\partial z} = \frac{\partial p}{\partial T}\partial_z T + \frac{\partial p}{\partial n}\partial_z n = -\rho g$$

b) *Constitutive* equation for the mass flux of intruder:

$$j_z = -\frac{m_0^2}{\rho} D_0 \partial_z n_0 - \frac{m_0 m}{\rho} D \partial_z n - \frac{\rho}{T} D^T \partial_z T$$

Non-convecting steady state $\longrightarrow j_z = 0$

Density gradients in terms of gravity and thermal gradient

$$\Lambda = \frac{\beta D^{T*} - (p^* + g^*)(D_0^* + D^*)}{\beta D_0^*}$$

$$p^* = p/nT, g^* = \rho g/n\partial_z T < 0$$

$$\beta = p^* + \phi\partial_\phi p^*, \phi = [\pi^{d/2}/2^{d-1}d\Gamma(d/2)]n\sigma^d$$

Determine the dependence of thermal diffusion on

$$\{m_0/m, \sigma_0/\sigma, \alpha, \alpha_0, \phi, g^*\}$$

Enskog kinetic theory: transport coefficients and equation of state

V. Garzó, J. Dufty, C. Hrenya PRE **76,** 0313303; 031304 (2007)

In particular, $p^* = p/nT = 1 + 2^{d-2}\chi\phi(1+\alpha)$

Pair correlation function

Explicit expressions for the transport coefficients

$$\{D_0^*, D^*, D^{T*}\}$$

Segregation criterion for BNE/RBNE $\longrightarrow$ $\boxed{\Lambda = 0}$

$$g^*(\gamma - M\beta) - \gamma\phi\frac{\partial p^*}{\partial \phi} + \frac{\omega^d}{2}\frac{M}{1+M}\chi_0\phi(1+\alpha_0)\left[(p^* + g^*)(1 + \frac{\gamma}{M})\Delta - \beta\right] = 0$$

$$\gamma = T_0/T, M = m_0/m, \omega = (\sigma+\sigma_0)/\sigma, \Delta = (\omega^{-d}/T\chi_0)(\partial_\phi \mu_0)_{T,n_0}$$

Special cases:

a) $m_0 = m, \sigma_0 = \sigma, \alpha = \alpha_0 \longrightarrow$ No segregation

b) Dilute gas ($\phi=0$)

$$\frac{T_0}{T} = \frac{m_0}{m}$$

J. Brey, M.J. Ruiz-Montero and F. Moreno, PRL **95**, 098001 (2005)
V. Garzó, EPL **75**, 521 (2006)

Due to the lack of energy equipartition, criterion is rather complicated since it involves all the parameter space *c)* Gravity dominates over thermal gradient $(|g^*| \to \infty)$

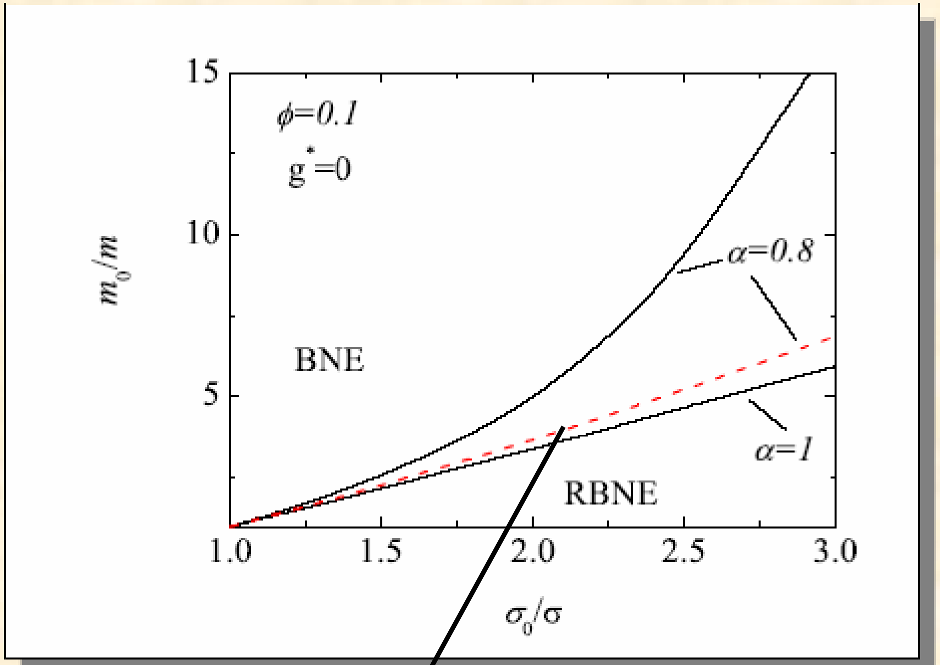
$$\frac{1+\frac{\omega^d}{2}\chi_0\phi(1+\alpha_0)\frac{\gamma+M}{1+M}\frac{\Delta}{\gamma}}{1+2^{d-2}\chi\phi(1+\alpha)\left[1+\phi\partial_\phi \ln(\phi\chi)\right]}\frac{mT_0}{m_0T}-1=0$$

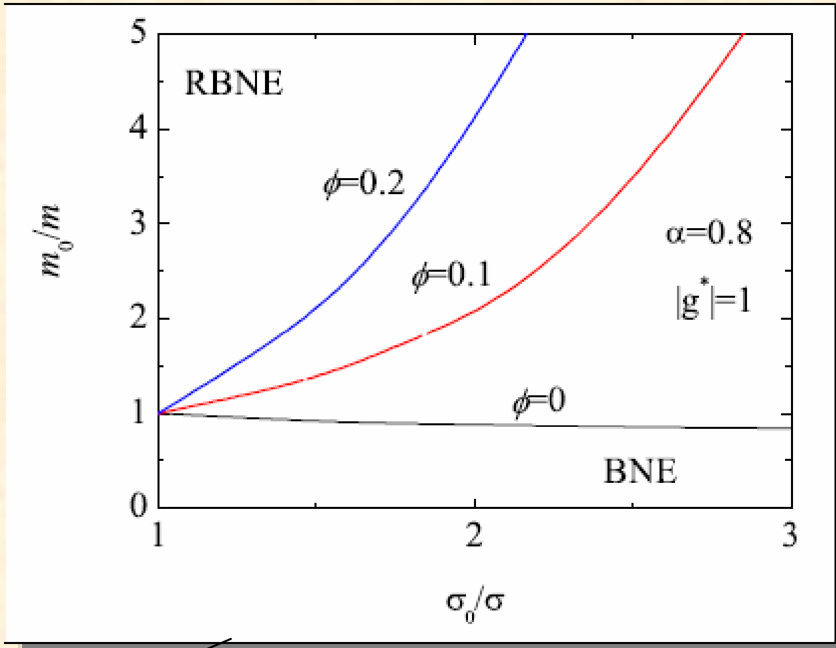
Different from $\frac{1+\frac{\omega^d}{2}\chi_0\phi}{1+2^{d-1}\chi\phi}\frac{mT_0}{m_0T}-1=0$

Jenkins and Yoon, PRL **88**, 194301 (2002);
Trujillo, Alam and Herrmann, EPL **64**, 190 (2003)

Both criteria agree under some approximations

Three-dimensional system (d=3), $\alpha=\alpha_0$

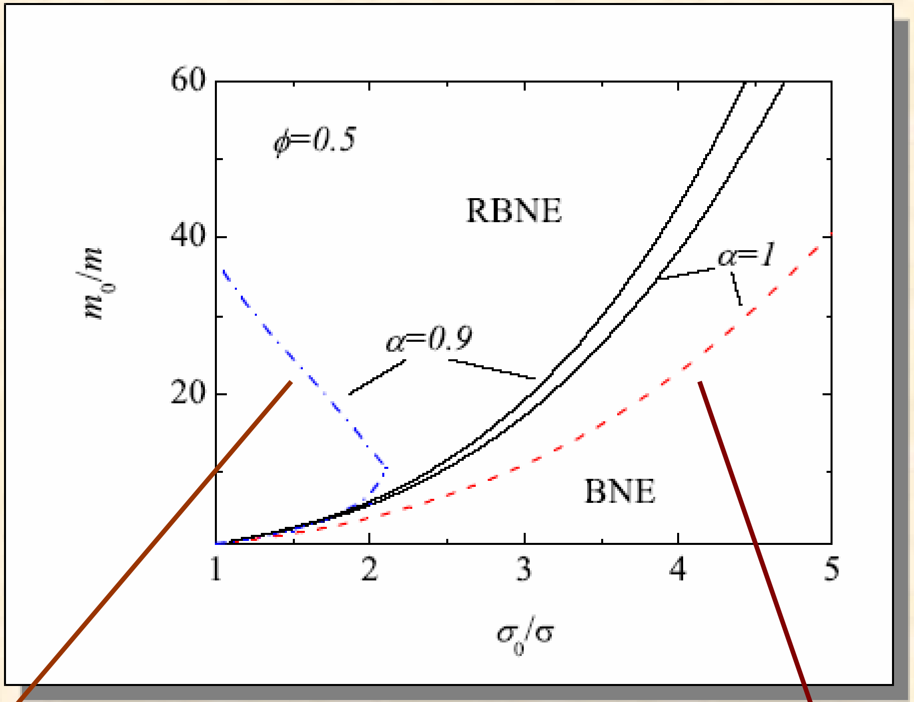




Galvin, Dahl &Hrenya,
JFM **528**, 207 (2005)

Similar trends observed in the experiments of Breu *et al*. PRL **90**, 014302 (2003)

$$|g^*| = \infty$$



Trujillo *et al*. EPL (2003)

Jenkins&Yoon, PRL (2002)

Positive aspects

•*Realistic theory* based on the **inelastic Enskog** kinetic theory to study ***segregation*** of an intruder in a moderately granular dense gas.

•*New* aspects covered by theory: finite density, combined effect of thermal gradients and gravity and not limited to weak dissipation

•*Consistent* with MD and experiments

Limitations

•*Tracer limit*. Extension to finite concentration of intruders

•*Vibrating* wall effect

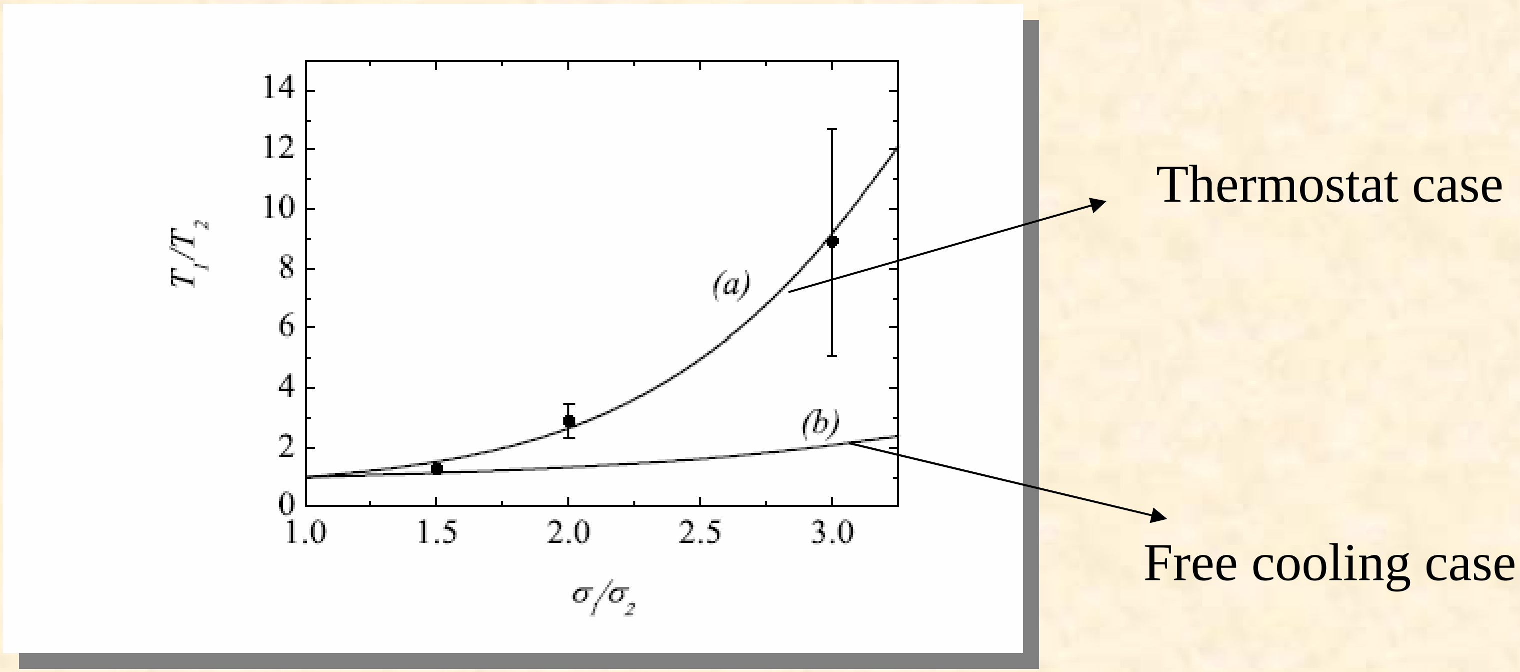
•Transport *linear* theory. Kinetic models

Work to be done……

Many thanks for your attention!!

Comparison with **experimental/simulation** results of Schroter, Ulrich, Kreft, Swift&Swinney (PRE **74**, 011307 (2006))

$$m_1/m_2 = (\sigma_1/\sigma_2)^3, x_2 = (\sigma_1/\sigma_2)^3 x_1, \alpha = 0.78$$



V. Garzó, EPL **75**, 521 (2006)