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Nonlinear transport in a binary mixture in the presence of gravitation

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Abstract

The effect of gravity on the tracer particles immersed in a dilute gas of mechanically different particles and subjected to the steady planar Couette flow is analyzed. The results are obtained from the Gross–Krook (GK) kinetic model of a binary mixture and the description applies for arbitrary values of both velocity and temperature gradients. The GK equation is solved by means of a perturbation method in powers of the field around a nonequilibrium state which retains all the hydrodynamic orders in the shear rate a and the thermal gradient ε. To first order in the gravity field, we explicitly determine the hydrodynamic profiles and the partial contributions to the momentum and heat fluxes associated with the tracer species. All these quantities are given in terms of a, ε, and the mass and size ratios. The shear-rate dependence of some of these quantities is illustrated for several values of the mass ratio showing that in general, the effect of gravity is more significant when the particles of the gas are lighter than the tracer particles. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

An usual assumption in studying the transport properties of gases under ordinary conditions is to ignore the presence of gravity. This is because the action of the field on a particle between two successive collisions is negligible, namely, $\ell \ll h$, where ℓ is

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the mean free path and $h = v_0^2/g$ is the characteristic distance associated with the gravity. Here, v_0 is the thermal velocity and g is the gravity acceleration. Thus, under usual laboratory conditions and at room temperature, $\ell/h \sim 10^{-11}$ so that the dependence of the irreversible fluxes on the gradients of the hydrodynamic fields is the same as in the case $g = 0$ [1–3]. Nevertheless, an interesting problem is to analyze if and how the transport equations are modified when the conditions of the rarefaction and/or the strength of the field are such that the ratio ℓ/h is not negligibly small. For instance, in the case of Earth's atmosphere, ℓ increases from 10^{-5} cm at the surface to tens of kilometers at an altitude of 500 km, while h changes in a range of 5–10 km up to an altitude of 100 km [4,5]. According to these orders of magnitude, the ratio ℓ/h fastly increases with the altitude so that $\ell/h \sim 1$ at the base of the exosphere [4].

The study of the influence of gravity on nonlinear transport has been a subject of interest for some of the present authors in the past few years. First, we evaluate the corrections to the Navier–Stokes equations due to gravity in a dilute gas under planar Fourier flow. This study was made from perturbation solutions of the Boltzmann equation (through order g^2) [6] and the Bhatnagar–Gross–Krook (BGK) kinetic model (through order g^6) [7]. We found that the heat flux is increased (decreased) with respect to its Navier–Stokes value when the gas is heated from above (below). These results present a good agreement with a finite difference numerical solution of the BGK equation [8] and with Monte Carlo simulations of the Boltzmann equation for a hard-sphere gas [9]. Recently, the above theoretical study has been extended to a more complex state than the Fourier flow, namely, the planar Couette flow. The results were obtained from the BGK model by performing a perturbation expansion in powers of the field around the pure (nonlinear) Couette flow, whose solution is known [10,11]. To first order in the gravity field we explicitly obtained the hydrodynamic profiles and the relevant nonlinear transport coefficients of the problem [12]. Given that the zeroth order approximation retains all the hydrodynamic orders in both the velocity and temperature gradients, the corresponding transport coefficients are nonlinear functions of both hydrodynamic gradients.

All the above results refer to a single gas. However, to the best of our knowledge, much less is known about the effect of gravity on transport in mixtures. Needless to say, the study of transport properties of these systems is much more complicated than that of a single gas. Not only is the number of transport coefficients much higher but also are functions of parameters such as the mass ratios, the molar fractions, and the size ratios. Due to the complexity of the general problem, it is instructive to consider first specific tractable situations where a thorough description may be offered. Perhaps, one of the simplest cases is the so-called *tracer* limit, namely, a binary mixture with a solute molar fraction negligibly small. This limit has the simplicity of the tagged particle problem but introduces the mass ratio as a new parameter into the dynamics of the problem. In this situation, one can assume that the state of the solvent (say, for instance species 2) is not practically disturbed by the presence of the tracer particles, while collisions among tracer species can be neglected in the kinetic equation for the velocity distribution function f_1 of the tracer particles. Thus, the distribution function

f_2 obeys a closed (nonlinear) Boltzmann equation and the distribution function f_1 a (linear) Boltzmann–Lorentz equation.

The aim of this paper is to analyze the effect of gravity on the tracer species immersed in a gas of mechanically different particles and subjected to the steady planar Couette flow. Since no solution of the Boltzmann equation is known for the planar Couette flow (even for a single gas), here we will use again the nonlinear kinetic model for mixtures proposed by Gross and Krook (GK) [13]. The reliability of this model to study nonlinear transport in mixtures has been shown in the past years in different nonequilibrium problems [14–16]. The fact that the GK model is constructed in the same spirit as the well-known BGK model [17] gives rise to the distribution f_2 verifies the BGK equation in the tracer limit, for which, as said before, a (perturbation) solution has been recently found [12]. Once the state of the solvent component 2 is well characterized, the goal is to determine the hydrodynamic profiles and the coefficients describing the transport of momentum and energy associated with the tracer particles. The problem is quite intricate and progress is possible here due to the previous work made in the absence of gravity [18]. Up to the first order in the field, the results show that the transport coefficients of the tracer species differ appreciably from those of the gas. In general, these coefficients are highly nonlinear functions of the mass and the size ratios and of the shear rate and the thermal gradient.

The plan of the paper is as follows. In Section 2 we introduce the model and describe the problem. Section 3 concerns with the perturbation solution of the GK model, considering the two first approximations. The new results refer to the first corrections of gravity to the hydrodynamic profiles and the fluxes. These corrections are obtained in Section 3, the mathematical details being given in the appendices. We complete the paper in Section 4 with some concluding remarks.

2. The kinetic model and the problem

Let us consider a binary mixture in the low-density regime described by the GK kinetic model [13] of the Boltzmann equation

$$\frac{\partial}{\partial t} f_1 + \mathbf{v} \cdot \nabla f_1 + \frac{\mathbf{F}_1}{m_1} \cdot \frac{\partial}{\partial \mathbf{v}} f_1 = -\nu_{11}(f_1 - f_{11}) - \nu_{12}(f_1 - f_{12}), \quad 1 \leftrightarrow 2 , \tag{1}$$

where f_i is the one-particle velocity distribution function of species i ($i \equiv 1,2$), $\mathbf{F}_i$ is an external force, ν_{ij} is an effective collision frequency, and f_{ij} is

$$f_{ij} = n_i \left(\frac{m_i}{2\pi k_B T_{ij}} \right)^{3/2} \exp\left(-\frac{m_i}{2k_B T_{ij}} (\mathbf{v} - \mathbf{u}_{ij})^2 \right) . \tag{2}$$

Here, m_i is the mass of a particle of species i, k_B is the Boltzmann constant, and we have introduced the fields

$$\mathbf{u}_{ij} = \frac{m_i \mathbf{u}_i + m_j \mathbf{u}_j}{m_i + m_j} , \tag{3}$$

$$T_{ij} = T_i + 2\frac{m_i m_j}{(m_i + m_j)^2}\left[(T_j - T_i) + \frac{m_i}{6k_B}(\mathbf{u}_i - \mathbf{u}_j)^2\right] . \tag{4}$$

Furthermore, the local number density, the mean velocity, and the partial temperature of species i are defined, respectively as

$$n_i = \int d\mathbf{v}\, f_i , \tag{5}$$

$$\mathbf{u}_i = \frac{1}{n_i}\int d\mathbf{v}\mathbf{v}\, f_i , \tag{6}$$

$$\frac{3}{2}n_i k_B T_i = \frac{m_i}{2}\int d\mathbf{v}(\mathbf{v} - \mathbf{u}_i)^2 f_i . \tag{7}$$

The collision terms of the GK model are obtained by requiring that their momentum and energy moments be the same as those of the Boltzmann operator for Maxwell molecules (i.e., an interaction potential of the form $\Phi_{ij} = \kappa_{ij} r^{-4}$). This allows one to identify v_{ij} as [13]

$$v_{ij} = A n_j\left[\kappa_{ij}\frac{m_i + m_j}{m_i m_j}\right]^{1/2} , \tag{8}$$

where $A = 4\pi \times 0.422$. Apart from the fields n_i, $\mathbf{u}_i$, and T_i, one can introduce the partial pressure tensor P_i (measuring the transport of momentum)

$$\mathsf{P}_i = m_i \int d\mathbf{v}(\mathbf{v} - \mathbf{u})(\mathbf{v} - \mathbf{u}) f_i \tag{9}$$

and the partial heat flux $\mathbf{q}_i$ (measuring the transport of energy)

$$\mathbf{q}_i = \frac{m_i}{2}\int d\mathbf{v}(\mathbf{v} - \mathbf{u})^2(\mathbf{v} - \mathbf{u}) f_i . \tag{10}$$

Here, $\mathbf{u} = (1/\rho)(\rho_1\mathbf{u}_1 + \rho_2\mathbf{u}_2)$ is the flow velocity of the mixture, $\rho = \rho_1 + \rho_2$, and $\rho_i = m_i n_i$ is the mass density of species i. Although the GK model can be extended to more general interactions [19], here, for the sake of concreteness, we will restrict ourselves to the case of Maxwell molecules.

We want to analyze the influence of gravity on the heat and momentum transport in a binary mixture. Since the general description of the nonlinear transport in a multi-component gas is a complex problem, we will choose a case that shares the simplicity of the single gas problem but yet incorporates the mass ratio as a new ingredient. This situation corresponds to the tracer limit, namely, when the molar fraction of one of the species is negligible. In this case, one expects that the state of the solvent component, say 2, is not disturbed by collisions with the tracer species, so that f_2 obeys a closed equation. In addition, since $n_1 \ll n_2$ one can neglect the collisions among tracer particles in the kinetic equation of f_1.

Under the above conditions, we assume that the gas (species 2) is driven away from equilibrium by the action of gravity and the presence of velocity and temperature gradients. The physical situation is that of a gas enclosed between two parallel plates in relative motion (planar Couette flow) and kept at different temperatures. In addition, the gas is also subjected to a constant gravitational field perpendicular to the plates. Let the x-axis be parallel to the direction of motion and the y-axis be normal to the plates. We want to analyze a steady state with velocity and temperature gradients along the y-direction coexisting with the fields $\mathbf{F}_i = -m_i g \hat{\mathbf{y}}$ $(i = 1, 2)$, where g is the acceleration due to gravity. It must be stressed that we are interested in a problem where the field does not generate convective motion so that the velocity profile is only due to boundary conditions. Therefore, in the tracer limit and under the geometry of the problem, the GK model reduces to

$$v_y \frac{\partial}{\partial y} f_1 - g \frac{\partial}{\partial v_y} f_1 = -\nu_{12}(f_1 - f_{12}) \tag{11}$$

for the tracer component and

$$v_y \frac{\partial}{\partial y} f_2 - g \frac{\partial}{\partial v_y} f_2 = -\nu_{22}(f_2 - f_{22}) \tag{12}$$

for the solvent component.

To simplify the analysis, it is convenient to introduce dimensionless quantities. To do that, we choose an *arbitrary* point y_0 belonging to the bulk domain as the origin and take the quantities at that point. Thus, we define $n_i^* \equiv n_i/n_0$, $T_i^* \equiv T_i/T_0$, $T_{ij}^* \equiv T_{ij}/T_0$, $p_i^* = (n_i k_B T_i)/n_0 k_B T_0 \equiv p_i/p_0$, $\mathbf{u}_i^* \equiv \mathbf{u}_i/v_0$, $\mathbf{u}_{ij}^* \equiv \mathbf{u}_{ij}/v_0$, $\mathbf{v}^* \equiv \mathbf{v}/v_0$, $f_i^* \equiv n_0^{-1} v_0^3 f_i$, $f_{ij}^* \equiv n_0^{-1} v_0^3 f_{ij}$, $\nu_{ij}^* \equiv \nu_{ij}/\nu_0$, and $g^* \equiv g/v_0 \nu_0$. Here, $v_0 = (k_B T_0/m_2)^{1/2}$ is the thermal velocity of the solvent particles and $\nu_0 = n_0(\nu_{22}/n_2)$. Further, we define the dimensionless fluxes $\mathsf{P}_i^* = \mathsf{P}_i/p_0$ and $\mathbf{q}_i^* = \mathbf{q}_i/p_0 v_0$.

In the above reduced units, Eq. (12) becomes

$$\left(1 + v_y^* \frac{\partial}{\partial s} - g^* \frac{p_2^*}{T_2^*} \frac{\partial}{\partial v_y^*}\right) f_2^* = f_{22}^* , \tag{13}$$

where the scaled variable s is

$$s = \frac{1}{v_0} \int_{y_0}^{y} \mathrm{d}y' \, \nu_{22}(y) . \tag{14}$$

Recently, Eq. (13) has been solved by means of a perturbation expansion in powers of gravity around the steady Couette flow [12]. Up to the first order in the field, the solution is characterized by the profiles

$$p_2^*(s) = 1 + \alpha_2 s \varepsilon g^* + \mathcal{O}(g^{*2}) , \tag{15}$$

$$u_{2,x}^*(s) = u_{2,x}^*(0) + as + \beta_2 s^2 \varepsilon g^* + \mathcal{O}(g^{*2}) , \tag{16}$$

$$T_2^*(s) = 1 + \varepsilon s - \gamma_{20} s^2 + (\gamma_{21} s^2 + \gamma_{22} s^3)\varepsilon g^* + \mathcal{O}(g^{*2}) , \tag{17}$$

where a (reduced shear rate) and ε (reduced thermal gradient) measure the departure of the system from equilibrium. The quantities α_2, β_2, γ_{20}, γ_{21}, and γ_{22} are highly *non-linear* functions of a and ε. Once the hydrodynamic fields are known, the momentum and heat fluxes can be obtained. Their explicit expressions are given in Ref. [12].

The task now is to solve the kinetic equation for the tracer species, Eq. (11), once the state of the solvent component is well known. This will be done in the next section.

3. Hydrodynamic profiles and transport properties of the tracer particles

In reduced units, the kinetic equation of f_1^* is given by

$$\left(1+\frac{v_y^*}{\tau}\frac{\partial}{\partial s}-g^*\frac{T_2^*}{\tau p_2^*}\frac{\partial}{\partial v_y^*}\right)f_1^*=f_{12}^*\,, \tag{18}$$

where

$$\tau=w^2\left(\frac{1+\mu}{2}\right)^{1/2}\,, \tag{19}$$

$$f_{12}^*=x_1(2\pi)^{-3/2}\,\bar{p}_{12}\bar{T}_{12}^{-5/2}\exp\left(-\frac{V^2}{2\bar{T}_{12}}\right)\,. \tag{20}$$

Here, $w\equiv(\kappa_{12}/\kappa_{22})^{1/4}$ and $\mu\equiv m_2/m_1$ are the force constant ratio and the mass ratio, respectively. Further, here, $x_1=n_1/n_2$ is the (constant) molar fraction, $\bar{p}_{12}=\mu p_{12}^*$, $p_{12}^*=n_2^*T_{12}^*$, and $\bar{T}_{12}=\mu T_{12}^*$. It is worthwhile to remark that although we are considering Maxwell molecules, an effective diameter σ_{ij} can be assigned to the interaction between particles of species i and j. Dimensional analysis allows one to interpret w as the size ratio σ_{12}/σ_{22}.

In the same way as in Eq. (12), the presence of gravity complicates the problem significantly, and consequently we look for again a perturbative solution in powers of gravity. The main feature of the expansion is that the reference state retains the full nonlinear dependence on the shear rate and the thermal gradient. In addition, given that the value of the gravity acceleration is small enough, for practical purposes it is sufficient to perform a perturbation analysis. Thus, we write

$$f_1^*=f_1^{(0)}+f_1^{(1)}\delta+\cdots\,, \tag{21}$$

where $\delta=\varepsilon g^*$. The use of δ instead of g^* as a perturbation parameter is due to the fact that the product εg^* appears in a natural way in the problem and the final expressions are more compact. We recall that the different approximations $f_1^{(k)}$ are nonlinear functions of the shear rate and the thermal gradient. The reference distribution function $f_1^{(0)}$ represents the steady Couette flow of the tracer species corresponding to the actual values of pressure, flow velocity, and both velocity and temperature gradients at the point of interest $y=y_0$. In a similar way, the fields p_{12}^*, $u_{12,x}^*$, and T_{12}^* as well as the dissipative fluxes must be expanded as

$$p_{12}^*=p_{12}^{(0)}+p_{12}^{(1)}\delta+\cdots\,, \tag{22}$$

$$u_{12,x}^*=u_{12,x}^{(0)}+u_{12,x}^{(1)}\delta+\cdots\,, \tag{23}$$

$$T_{12}^*=T_{12}^{(0)}+T_{12}^{(1)}\delta+\cdots\,, \tag{24}$$

$$\mathsf{P}_1^*=\mathsf{P}_1^{(0)}+\mathsf{P}_1^{(1)}\delta+\cdots\,, \tag{25}$$

$$\mathbf{q}_1^*=\mathbf{q}_1^{(0)}+\mathbf{q}_1^{(1)}\delta+\cdots\,. \tag{26}$$

Note that the different approximations $p_{12}^{(k)}$, $u_{12,x}^{(k)}$, and $T_{12}^{(k)}$ define the corresponding reference function $f_{12}^{(k)}$. By definition, $p_{12}^{(k)}(0)=u_{12,x}^{(k)}(0)=T_{12}^{(k)}(0)=\partial u_{12,x}^{(k)}/\partial s|_{s=0}=\partial T_{12,x}^{(k)}/\partial s|_{s=0}=0$ for $k \geqslant 1$. By substituting expansions (21)–(26) into Eq. (18), one gets a hierarchy of equations for the different distributions $f_1^{(k)}$. Now, we are going to examine the two first approximations.

3.1. Zeroth-order approximation

This approximation is concerned with a situation where gravity is zero. This reference state was analyzed by one of the present authors [18] and now we offer a brief summary of the main results. The zeroth-order distribution $f_1^{(0)}$ obeys the equation

$$\left(1+\frac{V_y}{\tau}\frac{\partial}{\partial s}\right) f_1^{(0)} = f_{12}^{(0)} , \tag{27}$$

where $\mathbf{V}=\mathbf{v}^* - \mathbf{u}_{12}^{(0)}$, $f_{12}^{(0)}=x_1\lambda\Phi_{12}$, with $\lambda \equiv \mu[(1-2M)\chi+2M]$, and

$$\Phi_{12}=(2\pi)^{-3/2}\bar{T}_{12}^{(0)-5/2}\exp\left(-\frac{V^2}{2\bar{T}_{12}^{(0)}}\right) . \tag{28}$$

Here, $\bar{T}_{12}^{(0)}=\mu T_{12}^{(0)}$, $M=\mu/(1+\mu)^2$, and $\chi=T_1^{(0)}/T_2^{(0)}$. The ratio χ between the temperatures of the tracer particles and the solvent component is the crucial quantity at this stage of description. Eq. (27) admits a solution [18] characterized by the absence of diffusion $\mathbf{u}_1^{(0)}=\mathbf{u}_{12}^{(0)}=\mathbf{u}_2^{(0)}$, a constant partial pressure $p_{12}^{(0)}$, and the profiles:

$$u_{12,x}^{(0)}(s)=u_{12,x}^*(0)+\tilde{a}\tau s , \tag{29}$$

$$\bar{T}_{12}^{(0)}(s)=\lambda+\tilde{\varepsilon}\tau s-\gamma_{10}\tau^2 s^2 . \tag{30}$$

Eqs. (29) and (30) imply that

$$\tilde{a}=\frac{a}{\tau}=\frac{a}{w^2[(1+\mu)/2]^{1/2}} , \tag{31}$$

$$\tilde{\varepsilon}=\frac{\lambda}{\tau}\varepsilon=\frac{\mu}{[(1+\mu)/2]^{1/2}}\frac{\chi+2M(1-\chi)}{w^2}\varepsilon , \tag{32}$$

$$\gamma_{10}=\frac{\lambda}{\tau^2}\gamma_{20}=\frac{2\mu}{1+\mu}\frac{\chi+2M(1-\chi)}{w^4}\gamma_{20} . \tag{33}$$

The self-consistency of the solution yields the implicit equation for γ_{10}:

$$2F_2(\gamma_{10})+\left(3-\frac{\tilde{a}^2}{\gamma_{10}}F_1(\gamma_{10})\right)=\frac{3}{\gamma_{10}}\frac{M(1-\chi)}{\chi+2M(1-\chi)} , \tag{34}$$

where we have introduced the functions

$$F_s(x)=\left[\frac{\mathrm{d}}{\mathrm{d}x}x\right]^s F_0(x) \tag{35}$$

and

$$F_0(x) = \frac{2}{x}\int_0^\infty \mathrm{d}t\, t\mathrm{e}^{-t^2/2} K_0(2x^{-1/4}t^{1/2}), \tag{36}$$

K_0 being the zeroth-order modified Bessel function.

From the above profiles, the momentum and heat fluxes of $f_1^{(0)}$ can be obtained. The nonzero elements of the pressure tensor and the heat flux are given by [18]

$$P^{(0)}_{1,xx} = x_1\frac{\lambda}{\mu}[1 + 4\gamma_{10}(F_1 + F_2)], \tag{37}$$

$$P^{(0)}_{1,yy} = x_1\frac{\lambda}{\mu}[1 - 2\gamma_{10}(F_1 + 2F_2)], \tag{38}$$

$$P^{(0)}_{1,zz} = x_1\frac{\lambda}{\mu}(1 - 2\gamma_{10}F_1), \tag{39}$$

$$P^{(0)}_{1,xy} = -x_1\frac{\lambda}{\mu\tau}F_0\frac{\partial}{\partial s}u^{(0)}_{12,x}, \tag{40}$$

$$q^{(0)}_{1,y} = -\frac{1}{2}x_1\frac{\lambda}{\mu\tau}\left[\frac{\tilde{a}^2}{\gamma_{10}}F_0 + \frac{3\mu M(1-\chi)}{\gamma_{10}\lambda}\right]\frac{\partial}{\partial s}\bar{T}^{(0)}_{12}, \tag{41}$$

$$q^{(0)}_{1,x} = x_1\frac{\lambda}{\mu\tau^2}[5F_2 + 2F_3 + 2\tilde{a}^2(F_2 + 5F_3 + 8F_4 + 4F_5)]\frac{\partial}{\partial s}\bar{T}^{(0)}_{12}. \tag{42}$$

Although the thermal gradient is only directed along the y-axis, Eq. (42) shows that the shear flow induces a nonzero x component of the heat flux. This effect is absent in the Navier–Stokes regime.

3.2. First-order approximation

As said in Section 1, the main goal of this paper is to assess the effects of gravity on the momentum and heat transport in a binary mixture. This can be done by computing the hydrodynamic fields and the fluxes to first order in δ. By substituting expansions (21)–(24) into Eq. (18), one gets

$$\left(1 + \frac{V_y}{\tau}\frac{\partial}{\partial s}\right) f_1^{(1)} = f_{12}^{(1)} + \frac{T_2^{(0)}}{\tau\varepsilon}\frac{\partial}{\partial V_y} f_1^{(0)}, \tag{43}$$

where $T_2^{(0)} = 1 + \varepsilon s - \gamma_{20}s^2$ and

$$f_{12}^{(1)} = x_1\left[\bar{p}_{12}^{(1)} + \lambda\frac{V_x u^{(1)}_{12,x}}{\bar{T}^{(0)}_{12}} + \lambda\left(\frac{V^2}{2\bar{T}^{(0)}_{12}} - \frac{5}{2}\right)\frac{\bar{T}^{(1)}_{12}}{\bar{T}^{(0)}_{12}}\right]\Phi_{12}. \tag{44}$$

Here, $\bar{p}_{12}^{(1)} = \mu p_{12}^{(1)}$ and $\bar{T}_{12}^{(1)} = \mu T_{12}^{(1)}$. Eq. (43) can be rewritten in the form

$$f_1^{(1)} = \left(1 + \frac{V_y}{\tau}\frac{\partial}{\partial s}\right)^{-1}\left(f_{12}^{(1)} + \frac{T_2^{(0)}}{\tau\varepsilon}\frac{\partial}{\partial V_y} f_1^{(0)}\right)$$
$$= \sum_{k=0}^{\infty}\left(-\frac{1}{\tau}\frac{\partial}{\partial s}\right)^k V_y^k\left(f_{12}^{(1)} + \frac{T_2^{(0)}}{\tau\varepsilon}\frac{\partial}{\partial V_y} f_1^{(0)}\right). \tag{45}$$

It is evident that this is only a formal solution since $f_{12}^{(1)}$ is a functional of $f_1^{(1)}$ through its dependence on the fields $p_{12}^{(1)}$, $u_{12,x}^{(1)}$, and $T_{12}^{(1)}$. For this reason, we need to determine the spatial dependence of the above fields to get the explicit expression of the distribution function. Guided by the results derived for the solvent component in the first order of gravity, Eqs. (15)–(17), we propose a similar solution for the tracer component. Specifically, inspection of Eq. (45) suggests that the hydrodynamic fields $\bar{p}_{12}^{(1)}$, $u_{12,x}^{(1)}$, and $\bar{T}_{12}^{(1)}$ are polynomials in the variable $\tilde{s} \equiv \tau s$ of degree 1, 2, and 3, respectively, whose coefficients are nonlinear functions of the reduced gradients a and ε and the parameters of the mixture w and μ. Therefore, we assume that

$$\bar{p}_{12}^{(1)} = \alpha_1 \tilde{s} \,, \tag{46}$$

$$u_{12,x}^{(1)} = \beta_1 \tilde{s}^2 \,, \tag{47}$$

$$\bar{T}_{12}^{(1)} = \gamma_{11} \tilde{s}^2 + \gamma_{12} \tilde{s}^3 \,. \tag{48}$$

The unknown coefficients α_1, β_1, γ_{11}, and γ_{12} must be obtained by requiring the self-consistency of solution (45) characterized by profiles (46)–(48), namely,

$$\int \mathrm{d}\mathbf{v}\{1, \mathbf{V}\}(f_1^{(1)} - f_{12}^{(1)}) = \{0, \mathbf{0}\} \,, \tag{49}$$

$$\int \mathrm{d}\mathbf{v}\, V^2 f_1^{(1)} = 3\,\bar{p}_1^{(1)} = \frac{3}{1-2M} x_1 [\bar{p}_{12}^{(1)} - 2M\,\bar{p}_2^{(1)}] \,, \tag{50}$$

with $\bar{p}_2^{(1)} = \mu p_2^{(1)}$. The accomplishment of these relations leads to a system of four linear equations (see Appendix A) for the set $\{\alpha_1, \beta_1, \gamma_{11}, \gamma_{12}\}$. The solution of this system provides explicit expressions of these coefficients in terms of a and ε as well as of w and μ. These expressions [c.f. Eqs. (A.11)–(A.16)] show a complex nonlinear dependence on the parameters of the problem. In particular, for vanishing shear rates, the above coefficients behave as

$$\alpha_1 \simeq -\frac{1}{\varepsilon\tau}(1 + 18\varepsilon\mu\tau\gamma_{12}^{(0)}) \,, \tag{51}$$

$$\beta_1 \simeq \tilde{a}\left(\frac{1}{2\varepsilon\mu\tau} - 18\gamma_{12}^{(0)}\right) \,, \tag{52}$$

$$\gamma_{11} \simeq \frac{1}{2\tau^2}\left(1 - \frac{828}{5}\varepsilon\mu\tau\gamma_{12}^{(0)}\right) \,, \tag{53}$$

$$\gamma_{12} \simeq \gamma_{12}^{(0)} \equiv \frac{1}{\varepsilon\tau}\frac{\mu - 1}{5 + 18\mu + 5\mu^2} \,. \tag{54}$$

In the case of mechanically equivalent particles ($w = 1$ and $\mu = 1$), $\alpha_1 = \alpha_2$, $\beta_1 = \beta_2$, $\gamma_{11} = \gamma_{21}$, and $\gamma_{22} = \gamma_{12}$, so that one recovers the results previously derived in the single gas [12]. However, when the mass ratio is different from one, the coefficients characterizing the hydrodynamic profiles of the gas and the tracer particles clearly differ. To illustrate this difference, in Fig. 1 we show the shear-rate dependence of the (reduced)

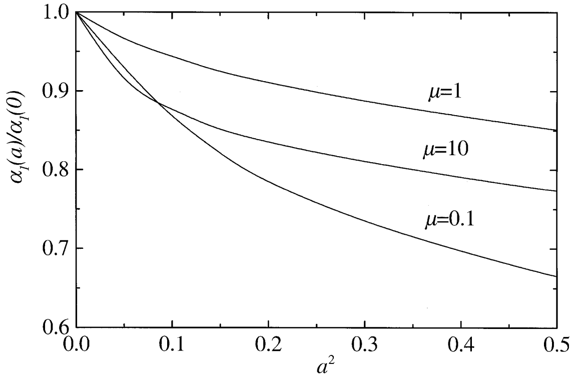

Fig. 1. Shear-rate dependence of the reduced coefficient $\alpha_1(a)/\alpha_1(0)$ for $w=1$, $\varepsilon=1$, and three different values of the mass ratio $\mu=m_2/m_1$: $\mu=0.1$, 1, and 10.

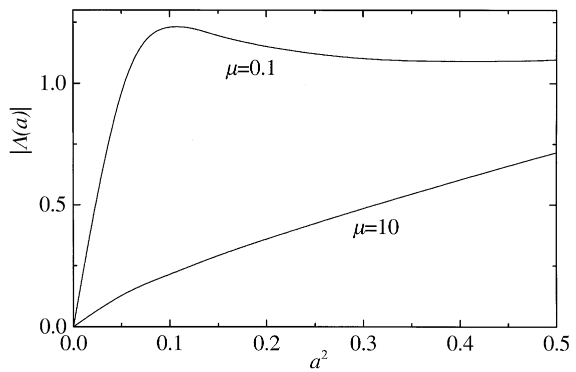

Fig. 2. Shear-rate dependence of the magnitude of the coefficient $\Lambda(a)$ for $w=1$, $\varepsilon=1$, and two different values of the mass ratio $\mu=m_2/m_1$: $\mu=0.1$ and 10.

coefficient $\alpha_1(a)/\alpha_1(0)$ for $\omega=1$ and three different values of the mass ratio $\mu=0.1, 1$, and 10. In general, the magnitude of α_1 decreases as the shear rate increases whatever the mass ratio considered is. We see that the influence of the mass ratio on the behavior of α_1 is quite significant, especially when the tracer particles are heavier than the gas particles (Brownian limit). A particularly interesting coefficient is β_1. It can be seen as a measure of the diffusion of the tracer particles induced by the coupling between the shear flow and gravity (tracer-particle sedimentation). In reduced units, the tracer-particle current density measured in the reference frame in which the gas is locally at rest is

$$u^*_{1,x}(s) - u^*_{2,x}(s) = (\beta_1\tau^2 - \beta_2)s^2\varepsilon g^* \equiv \Lambda(a)s^2\varepsilon g^* \ , \tag{55}$$

where the "transport" coefficient $\Lambda=\beta_1\tau^2-\beta_2$ can be identified as the "mobility" coefficient of the infinitely diluted suspension. When $\mu=1$ and/or $a=0$, $\Lambda=0$ and no diffusion appears in the system. In Fig. 2, we plot the magnitude of $\Lambda(a)$ for $\omega=1$ and

$\mu = 0.1$ and 10. In general, for nonzero shear rates, this coefficient is clearly different from zero for disparate masses. While $|\Lambda|$ increases with the shear rate when $\mu > 1$, the coefficient Λ reaches a maximum in the case of $\mu < 1$.

Once the hydrodynamic profiles have been obtained, the next step is to get the tracer contribution to the momentum and heat transport across the system when only terms through first order in gravity are taken into account. Since the reduced gradients a and ε are defined at the (arbitrary) point $s = 0$, we compute the fluxes at that point, without loss of generality. In the same way as in our previous work on the single gas case [12], the idea is to express the momentum and heat fluxes at *any* point in the bulk region in terms of the values of the hydrodynamic quantities and their gradients at that very point. It is convenient to define a (dimensionless) partial generalized shear viscosity η_1 and a (dimensionless) partial generalized thermal conductivity κ_1 through the relations

$$\eta_1(a, \varepsilon, \delta) = - \left. \frac{P_{1,xy}^{(0)} + P_{1,xy}^{(1)}\delta}{x_1 a} \right|_{s=0} , \tag{56}$$

$$\kappa_1(a, \varepsilon, \delta) = - \left. \frac{q_{1,y}^{(0)} + q_{1,y}^{(1)}\delta}{x_1 \varepsilon} \right|_{s=0} . \tag{57}$$

In addition, due to the anisotropy of the problem, a new generalized transport coefficient Φ_1 is defined through the x component of the heat flux:

$$\Phi_1(a, \varepsilon, \delta) = - \left. \frac{q_{1,x}^{(0)} + q_{1,x}^{(1)}\delta}{x_1 \varepsilon} \right|_{s=0} . \tag{58}$$

Eqs. (56)–(58) define the transport coefficients measuring the contribution of the tracer component to the momentum and heat fluxes of the mixture. The explicit expressions of $P_{1,xy}^{(1)}$, $q_{1,y}^{(1)}$, and $q_{1,x}^{(1)}$ are displayed in Appendix B. For zero shear rates, $\Phi_1 = 0$ and η_1 and κ_1 behave, respectively, as

$$\eta_1 \simeq \frac{1}{\tau} + \frac{8}{\tau^3} \left[1 - \frac{9}{10} \varepsilon \tau^2 \gamma_{12}^{(0)} (279\mu - 500) \right] \delta , \tag{59}$$

$$\kappa_1 \simeq \frac{5}{2\tau} + \frac{9\tau\gamma_{12}^{(0)}(7074\varepsilon^2\mu^2 + 70\varepsilon\tau^2 - 25\tau^2) + 145\varepsilon\mu}{5\varepsilon\tau^3} \delta . \tag{60}$$

Given that our results are restricted to the first order in the field, we do not have the complete dependence of the generalized transport coefficients on the gravity field. In an attempt to show the main trends expected in the presence of nonzero gravity, we plot the partial shear viscosity η_1 as a function of the shear rate a for $w = 1$, $\varepsilon = 1$ (points heated from "above") and $\mu = 0.1$ and 10. We consider $g^* = 0$ and 0.002. The coefficient η_1 is the most important transport coefficient of the Couette flow problem. Fig. 3 shows how the presence of the field does not change the trends observed for η_1 in the absence of gravitation. In addition, the effect of the field on the momentum transport is more significant when the tracer species are heavier than the solvent component. On the other hand, the partial shear viscosities with and without gravitational field tend

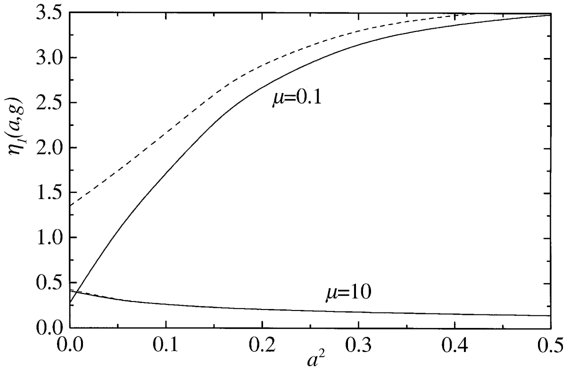

Fig. 3. Shear-rate dependence of the partial shear viscosity η_1 for $w = 1$, $\varepsilon = 1$, and two different values of the mass ratio $\mu = m_2/m_1$: $\mu = 0.1$ and 10. Two values of gravity have been considered: $g^* = 0.002$ (solid lines) and $g^* = 0$ (dashed lines).

to overlap as a increases whatever the mass ratio considered is. In fact, the effect of gravity on η_1 is practically negligible in the limit of large shear rates.

4. Concluding remarks

The primary objective of this paper has been to analyze the influence of gravitation on the hydrodynamic fields and the partial transport coefficients of the tracer particles immersed in a dilute gas under Couette flow. The starting point to describe the state of the mixture is the GK kinetic model for Maxwell molecules. In the tracer limit, the GK equation for the velocity distribution function of the gas f_2 reduces to a closed BGK equation, for which a solution through first order in the field g has been recently obtained [12]. The corresponding GK equation for the velocity distribution function f_1 of tracer particles is also solved by means of a perturbation expansion in powers of g. The reference state of this expansion is an exact solution of the GK equation for arbitrary values of the shear rate and the thermal gradient [18]. Although our calculations have been restricted to the first-order correction, for practical purposes this approximation should be sufficient. Given that our reference state contains all the hydrodynamic orders in both the (reduced) shear rate a and the (reduced) thermal gradient ε, the corresponding transport coefficients associated with the tracer species are highly nonlinear functions of a and ε. In addition, no restriction on the mass ratio and the force constant ratio is considered in our solution.

In the same way as for the solvent component [12], our results show that the partial pressure $p_1^{(1)}$, tracer flow velocity $u_{1,x}^{(1)}$, and the partial temperature $T_1^{(1)}$ become linear, quadratic, and cubic functions, respectively, with respect to a conveniently scaled space variable. Since $u_{2,x} \neq u_{1,x}$, a tracer diffusion process appears in the mixture. The corresponding mass current is due to the coupling between the gravity and the shear flow. In fact, it disapears when $a = 0$ and/or $g = 0$. This process can be of practical interest in sedimentation problems, for example. The corresponding transport coefficient Λ characterizing the diffusion has a highly nonlinear dependence on the parameters of the problem, especially on a and the mass ratio μ. Once the hydrodynamic profiles are known, we have determined the partial contributions to the momentum and heat fluxes coming from the tracer species. Their explicit first-order corrections are displayed in Appendix B. As an illustration, we have analyzed with more detail the partial shear viscosity η_1 and the results show that the effect of gravity is more significant as the tracer particles are heavier than the solvent particles. Nevertheless, the influence of g on η_1 tends to decrease as the shear rate becomes large.

Although the results reported in this paper have been obtained from the GK model, we expect that the main trends observed here will be present in the context of the Boltzmann equation. Since an exact solution of this equation in the Couette flow problem is unapproachable, an alternative to solve numerically is the use of the direct simulation Monte Carlo method [20]. Work along this line is in progress. On the other hand, we are fully aware that the tracer limit is certainly a limitation that one would like to get rid of. In this sense, we expect that considering again the GK model we will be able to analyze the more general problem when the molar fractions of both components are arbitrary. A recent solution [21] of the GK model for a multicomponent mixture under Couette flow could be the starting point to study the effect of gravity in the general case.

Acknowledgements

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Appendix A. Consistency of the solution

This appendix is concerned with the fulfillment of the consistency conditions (49) and (50). To do that, it is convenient to rewrite the formal solution (45) as

$$f_1^{(1)} = \sum_{k=0}^{\infty} \left(-\frac{\partial}{\partial \tilde{s}} \right)^k \left[f_{12}^{(1)} + x_1 \frac{\bar{T}_{12}^{(0)}}{\varepsilon\tau} \frac{\partial}{\partial V_y} \Phi_1^{(0)} \right] , \tag{A.1}$$

where $\Phi_1^{(0)}$ is the formal solution in the absence of gravitation [18]:

$$\Phi_1^{(0)} = \sum_{\ell=0}^{\infty} \left(-\frac{\partial}{\partial \tilde{s}} \right)^{\ell} V_y^{\ell} \Phi_{12} . \tag{A.2}$$

From a formal point of view, the velocity moments of $f_1^{(1)}$ can be directly obtained from the velocity moments of $f_2^{(1)}$ when one exploits the equivalence between the kinetic equation (43) and its corresponding counterpart of $f_2^{(1)}$. Consequently, the moments of $f_1^{(1)}$ can be determined from comparison with those of $f_2^{(1)}$ by making the changes: $s \to \tilde{s}$, $a \to \tilde{a}$, $\gamma_{20} \to \gamma_{10}$, $\alpha_2 \to \alpha_1$, $\beta_2 \to \lambda\beta_1$, $\gamma_{21} \to \lambda\gamma_{11}$, and $\gamma_{22} \to \lambda\gamma_{12}$. Taking into account this equivalence and the results quoted in Appendices A and B of Ref. [12], we can explicitly write the first few moments of the distribution $f_1^{(1)}$.

The condition for the density, first relation of Eq. (49), is trivially satisfied. The condition for the x component of the flow velocity implies that

$$2F_1(\lambda\beta_1 + \tilde{a}\alpha_1) - 2\tilde{a}(F_2 - F_1)\lambda\frac{\gamma_{12}}{\gamma_{10}} + \frac{\tilde{a}}{\varepsilon\tau}\left[1 - \gamma_{10}\left(4F_4 + \frac{28}{3}F_3 + 7F_2 + \frac{5}{3}F_1\right)\right] = 0\,. \tag{A.3}$$

Here, we have used the asymptotic expansion of the function $F_r(\gamma_{10})$ defined by Eqs. (35) and (36) [10,11]:

$$F_r \equiv F_r(\gamma_{10}) = \sum_{k=0}^{\infty}(k+1)^r(2k+1)!(2k+1)!!(-\gamma_{10})^k\,. \tag{A.4}$$

The condition for the y component of the flow velocity leads to the equation

$$\alpha_1F_0 - \lambda\frac{\gamma_{12}}{\gamma_{10}}(F_1 - F_0) + \frac{1}{\varepsilon\tau}\left[1 - \gamma_{10}(2F_3 + 3F_2 + F_1)\right] = 0\,. \tag{A.5}$$

Let us consider now the consistency condition for the pressure, Eq. (50). Taking into account the results of Ref. [12] and after some algebra, this condition can be cast into the form

$$3\frac{2M}{1-2M}\left(\frac{\alpha_1}{2M} - \frac{\mu\alpha_2}{\tau}\right)\tilde{s} = C\tilde{s} + D\,, \tag{A.6}$$

where

$$C = 3\alpha_1 - 2\alpha_1\gamma_{10}(4F_3 + 8F_2 + 3F_1) + 2\lambda\gamma_{12}(4F_4 + 8F_3 + 3F_2) + 8\tilde{a}\lambda\beta_1F_2 + 2\tilde{a}^2\left[\alpha_1(2F_2 - F_1) - \lambda\frac{\gamma_{12}}{\gamma_{10}}(2F_3 - 3F_2 + F_1)\right] - 2\frac{\gamma_{10}}{\varepsilon\tau}\Delta\,, \tag{A.7}$$

$$D = 2\lambda\left(\frac{\alpha_1\varepsilon}{\tau} + \gamma_{11}\right)(3F_2 + 2F_3) + 4\lambda^2\frac{\gamma_{12}\varepsilon}{\tau}(8F_7 + 44F_6 + 86F_5 + 77F_4 + 32F_3 + 5F_2) + 8\tilde{a}\lambda^2\frac{\beta_1\varepsilon}{\tau}(4F_5 + 8F_4 + 5F_3 + F_2) - 2\lambda\frac{\tilde{a}^2}{\gamma_{10}}\left(\frac{\alpha_1\varepsilon}{\tau} + \gamma_{11}\right)(F_2 - F_1) + 2\tilde{a}^2\frac{\lambda^2\varepsilon}{\tau}\frac{\gamma_{12}}{\gamma_{10}^2}(F_3 - 3F_2 + 2F_1) + \frac{\lambda}{\tau^2}\Delta\,. \tag{A.8}$$

In these expressions, we have introduced the shear-rate dependent quantity

$$\Delta(\tilde{a}) = 2 + \frac{5}{3}F_1 + \frac{4}{3}F_2 - \frac{\gamma_{10}}{3}(24F_6 + 124F_5 + 222F_4 + 185F_3 + 75F_2 + 12F_1) + \frac{2}{3}\tilde{a}^2(8F_4 + 18F_3 + 13F_2 + 3F_1) - \frac{2}{3}\tilde{a}^2\gamma_{10}(48F_8 + 352F_7 + 1024F_6 + 1500F_5 + 1157F_4 + 443F_3 + 66F_2)\,. \tag{A.9}$$

Since relation (A.6) must be verified at any point, the pressure condition requires that

$$\frac{2M}{1-2M}\left(\frac{\alpha_1}{2M} - \frac{\mu\alpha_2}{\tau}\right) = \frac{1}{3}C, \quad D = 0\,. \tag{A.10}$$

The solution of the system of algebraic equations (A.4), (A.5), and (A.10) provides the explicit expressions of the coefficients α_1, β_1, γ_{11}, and γ_{12} of the hydrodynamic profiles. The solution for γ_{12} can be written as

$$\gamma_{12} = -\frac{\gamma_{10}}{\lambda\varepsilon\tau}\frac{R}{S}\,, \tag{A.11}$$

where the coefficients R and S are given by

$$R = -3\mu M\alpha_2\varepsilon F_0F_1 + (2M-1)\left\{\frac{\tilde{a}^2}{3}\gamma_{10}[2F_0F_2(5F_1 + 21F_2 + 28F_3 + 12F_4) - 3F_1(F_1 + 2F_2)(F_1 + 3F_2 + 2F_3)] - (2F_0F_2 - F_1^2 - 2F_1F_2)\tilde{a}^2 - F_0F_1\gamma_{10}\Delta\right\} - F_1[\gamma_{10}(F_1 + 3F_2 + 2F_3) - 1][\gamma_{10}(3F_1 + 8F_2 + 4F_3)(2M-1) - 3M]\,, \tag{A.12}$$

$$S = (2M-1)\tilde{a}^2\left\{F_0[F_1(F_2 - 2F_3) + 4F_2^2] - F_1^2(F_1 + 2F_2)\right\} + (2M-1)\gamma_{10}F_1\left\{F_0(4F_4 + 12F_3 + 11F_2 + 3F_1) - F_1(3F_1 + 8F_2 + 4F_3)\right\} - 3MF_1(F_0 - F_1)\,. \tag{A.13}$$

The remaining coefficients are

$$\alpha_1 = \frac{1}{\varepsilon\tau F_0}\left[\gamma_{10}(F_1 + 3F_2 + 2F_3) + \varepsilon\lambda\tau(F_1 - F_0)\frac{\gamma_{12}}{\gamma_{10}} - 1\right]\,, \tag{A.14}$$

$$\beta_1 = -\tilde{a}\left[\frac{\alpha_1}{\lambda} - \frac{\gamma_{10}}{6\varepsilon\lambda\tau}\frac{5F_1 + 21F_2 + 28F_3 + 12F_4}{F_1} - \frac{\gamma_{12}}{\gamma_{10}}\frac{F_2 - F_1}{F_1} + \frac{1}{2\varepsilon F_1\tau\lambda}\right]\,, \tag{A.15}$$

$$\gamma_{11} = -\frac{\alpha_1\varepsilon}{\tau} - \frac{1}{\tau[\tilde{a}^2(F_1 - F_2) + \gamma_{10}(3F_2 + 2F_3)]}\left\{4\tilde{a}\varepsilon\gamma_{10}\lambda\beta_1(F_2 + 5F_3 + 8F_4 + 4F_5) + \varepsilon\lambda\frac{\gamma_{12}}{\gamma_{10}}[\tilde{a}^2(2F_1 - 3F_2 + F_3) + 2\gamma_{10}^2(8F_7 + 44F_6 + 86F_5 + 77F_4 + 32F_3 + 5F_2)] + \frac{\gamma_{10}}{2\tau}\Delta\right\}\,. \tag{A.16}$$

Eqs. (A.11)–(A.16) show the highly nonlinear dependence of the hydrodynamic profiles on the shear rate and the mass ratio. On the other hand, the dependence on the reduced thermal gradient is quite simple since α_1, β_1, and γ_{12} are inversely proportional to ε while γ_{11} does not depend on ε. In the case of mechanically equivalent particles ($w=1$ and $\mu=1$), all the above expressions reduce to those previously obtained in the single gas case [12].

Appendix B. Pressure tensor and heat flux

In this appendix we get the generalized transport coefficients η_1 (Eq. (56)), κ_1 (Eq. (57)), and Φ_1 (Eq. (58)). The explicit expressions of these coefficients can be obtained from the fluxes $P^{(1)}_{1,xy}$, $q^{(1)}_{1,y}$, and $q^{(1)}_{1,x}$, respectively. These fluxes can be computed by following identical mathematical steps as those made in Appendix A. The results are

$$
\begin{aligned}
P^{(1)}_{1,xy}|_{s=0} &= \mu^{-1}\int \mathrm{d}\mathbf{v}\, V_x V_y f_1^{(1)} \\
&= x_1\frac{\lambda}{\mu\tau}\frac{\beta_1\lambda\varepsilon+\tilde{a}(\alpha_1\varepsilon+\tau\gamma_{11})}{\gamma_{10}}(F_1-F_0)-x_1\tilde{a}\frac{\lambda\varepsilon}{\mu\tau}\frac{\gamma_{12}}{\gamma_{10}^2}(F_2-3F_1+2F_0) \\
&\quad + x_1\frac{\tilde{a}\lambda}{\mu\tau^2}\left[2\gamma_{10}(8F_7+44F_6+90F_5+85F_4+37F_3+6F_2)\right. \\
&\quad \left. -\frac{4}{3}(2F_3+3F_2+F_1)\right], \qquad \text{(B.1)}
\end{aligned}
$$

$$
\begin{aligned}
q^{(1)}_{1,y}|_{s=0} &= \frac{1}{2\mu}\int \mathrm{d}\mathbf{v}\, V^2 V_y f_1^{(1)} \\
&= -x_1\frac{\alpha_1}{2\mu}(5+18\tilde{a}^2)-18\frac{x_1\lambda}{\mu}\gamma_{12}(7+100\tilde{a}^2) \\
&\quad - x_1\frac{\lambda\beta_1\tilde{a}}{\mu}\left[2+\frac{\lambda^2\varepsilon^2}{2\tau^2\gamma_{10}^2}(F_2-3F_1+2F_0)\right] \\
&\quad + x_1\frac{\lambda^2\varepsilon}{4\mu\tau^2\gamma_{10}}(\alpha_1\varepsilon+2\tau\gamma_{11})[2F_3+F_2-3F_1+2\tilde{a}^2(4F_4-3F_2-F_1)] \\
&\quad - x_1\frac{\lambda^3\varepsilon^2}{4\mu\tau^2}\frac{\gamma_{12}}{\gamma_{10}^2}[2F_4-3F_3-5F_2+6F_1 \\
&\quad + 2\tilde{a}^2(4F_5-8F_4-3F_3+5F_2+2F_1)] \\
&\quad - x_1\frac{3+2F_1}{\mu\tau\varepsilon}+x_1\frac{\gamma_{10}}{\mu\tau\varepsilon}(2F_3+5F_2+3F_1+17)
\end{aligned}
$$

$$
\begin{aligned}
&- x_1 \frac{\tilde{a}^2}{\mu\tau\varepsilon}\left(F_3 + \frac{3}{2}F_2 + \frac{1}{2}F_1 - 270\right) \\
&- x_1 \frac{\lambda^2\varepsilon}{\mu\tau^3}\left(F_6 + 6F_5 + \frac{47}{4}F_4 + \frac{25}{3}F_3 + \frac{23}{12}F_2\right) \\
&- x_1 \tilde{a}^2 \frac{\lambda^2\varepsilon}{\mu\tau^3}\left(4F_8 + 26F_7 + 67F_6 + \frac{175}{2}F_5 + 61F_4 + \frac{43}{2}F_3 + 3F_2\right),
\end{aligned}
\tag{B.2}
$$

$$
\begin{aligned}
& q^{(1)}_{1,x}|_{s=0} \\
&= \frac{1}{2\mu}\int \mathrm{d}\mathbf{v}\, V^2 V_x f_1^{(1)} \\
&= x_1 \frac{\tilde{a}\alpha_1}{\mu\gamma_{10}}\left\{\frac{\lambda^2\varepsilon^2}{\tau^2}\left[\tilde{a}^2(F_2 + 4F_3 + 3F_4 - 4F_5 - 4F_6) + \frac{1}{2}(5F_2 - 3F_3 - 2F_4)\right]\right. \\
&\quad \left. + \gamma_{10}[\tilde{a}^2(8F_5 + 16F_4 + 10F_3 + 2F_2) + 2F_3 + 5F_2]\right\} \\
&\quad + x_1 \frac{\lambda\beta_1}{2\mu\gamma_{10}^2}\left\{\frac{\lambda^2\varepsilon^2}{\tau^2}[3\tilde{a}^2(F_3 - 3F_2 + 2F_1) + \gamma_{10}(5F_2 - 3F_3 - 2F_4)]\right. \\
&\quad \left. + \gamma_{10}[6\tilde{a}^2(F_1 - F_2) + 2\gamma_{10}(2F_3 + 5F_2)]\right\} \\
&\quad + x_1 \frac{\tilde{a}\lambda^2\varepsilon}{\mu\tau}\frac{\gamma_{11}}{\gamma_{10}}[2\tilde{a}^2(F_2 + 4F_3 + 3F_4 - 4F_5 - 4F_6) + 5F_2 - 3F_3 - 2F_4] \\
&\quad + x_1 \frac{\tilde{a}\lambda}{2\mu}\frac{\gamma_{12}}{\gamma_{10}^2}\left\{\frac{\lambda^2\varepsilon^2}{\tau^2}[2\tilde{a}^2(2F_2 + 7F_3 + 2F_4 - 11F_5 - 4F_6 + 4F_7)\right. \\
&\quad + 10F_2 - 11F_3 - F_4 + 2F_5] \\
&\quad \left. + \gamma_{10}[2\tilde{a}^2(6F_4 + 8F_3 + 2F_2 - 8F_5 - 8F_6) + 10F_2 - 6F_3 - 4F_4]\right\} \\
&\quad + x_1 \frac{\tilde{a}}{2\tau\mu\varepsilon}(2F_2 + 3F_1 + 2F_0) \\
&\quad - x_1 \frac{\tilde{a}\gamma_{10}}{6\tau\mu\varepsilon}(24F_6 + 148F_5 + 246F_4 + 155F_3 + 33F_2) \\
&\quad + x_1 \frac{\tilde{a}^3}{\tau\mu\varepsilon}(4F_4 + 8F_3 + 5F_2 + F_1) \\
&\quad - x_1 \frac{\tilde{a}^3\gamma_{10}}{3\tau\mu\varepsilon}(48F_8 + 352F_7 + 1024F_6 + 1500F_5 + 1157F_4 + 443F_3 + 66F_2) \\
&\quad - x_1 \frac{\lambda^2\varepsilon}{\mu\tau^3}\left[\frac{\tilde{a}}{12}(33F_2 + 122F_3 + 91F_4 - 98F_5 - 124F_6 - 24F_7)\right.
\end{aligned}
$$

$$+\frac{\tilde{a}^3}{6}(66F_2+377F_3+714F_4+343F_5-476F_6-672F_7-304F_8-48F_9)\Bigg]$$

$$+x_1\frac{\lambda^3\varepsilon^2}{3\mu\tau^4}[\tilde{a}(16F_6+100F_5+164F_4+101F_3+21F_2)$$

$$+\tilde{a}^3(64F_8+464F_7+1336F_6+1940F_5+1486F_4+566F_3+84F_2)]\,. \tag{B.3}$$

From Eqs. (40)–(42) and (B.1)–(B.3) one can identify the explicit expressions of the transport coefficients.

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